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2. Algebra

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Spetses

Weyl groups are finite groups acting as a reflection group on a rational vector space. Starting with the classification of finite-dimensional semisimple complex Lie algebras in terms of their Weyl groups, these rational reflection groups made their appearance as the skeleton of many more algebraic objects, like algebraic groups, finite groups of Lie type, quantum groups, Iwahori-Hecke algebras, braid groups, ... governing much of the structure and of the properties of these objects.
By extension of base field, Weyl groups are in particular finite complex reflection groups. In this talk we report on recent results and conjectures about analogues of some of the above structures with 'Weyl group' a complex reflection group. It was first noted by Springer that non-real reflection groups naturally appear inside Weyl groups as what is now called relative Weyl groups, that is, normalizers modulo centralizers of subspaces. The importance of this construction was revealed in the work of Broué, Michel and the author on the ℓ-blocks of characters of finite groups of Lie type. There it was shown that the unipotent characters inside an ℓ-block are parametrized by the irreducible characters of a relative Weyl group, which in general is a non-real reflection group. A possible interpretation of this result was subsequently proposed by Broué and the author in terms of the so-called cyclotomic Hecke algebra attached to a finite complex reflection group. This generalizes the usual Iwahori-Hecke algebra of real reflection groups.
The results of Lusztig show that the set of unipotent characters of a finite group of Lie type only depends on the Weyl group of the associated algebraic group, together with the action of the Frobenius endomorphism on it. Moreover, this set carries a rich combinatorial structure. In the course of this classification Lusztig observed that similar sets can formally also be attached to those finite real reflection groups which are not Weyl groups, just as if there existed an algebraic group whose Weyl group is non-crystallographic. On a conference on the Greek island Spetses it was first proposed by Michel Broué that a similar construction might also exist for non-real reflection groups. This was shown to be true for all but finitely many types of irreducible reflection groups by constructing sets of 'unipotent characters', Fourier transform matrices, and eigenvalues of Frobenius,

which satisfy all the combinatorial properties of unipotent characters of actual finite groups of Lie type.

The central object in this construction is the cyclotomic Hecke algebra of a complex reflection group W. This can be defined by generators and relations, as in the case of real reflection groups. Broué, Rouquier and the author gave a more intrinsic construction as a quotient of the braid group of W, i. e., of the fundamental group of the space of regular orbits of W on its natural vector space.

It is tempting to speculate about an underlying algebraic structure giving rise to the unipotent characters attached to complex reflection groups. We would like to call such a conjectural object 'spets', because it was first dreamed of in Spetses and should possess such 'spetsial' properties. Until now, such an object has not been found, but a lot of intriguing evidence for its existence has been given.