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Abstracts of Plenary and Invited Lectures

Section:

2. Algebra

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The abelian defect group conjecture

Let G be a finite group, k an algebraically closed field of prime characteristic p, and B a block of the group algebra kG with positive defect (i. e., a block that is not just a matrix ring over k). Over 10 years ago, Alperin formulated his famous 'Weight Conjecture', which states that the number of isomorphism classes of B-modules can be calculated 'locally' (i. e., in terms of the representation theory of normalizers of non-trivial p-subgroups of G) using an explicit formula. This conjecture subsumed several earlier conjectures on the number of simple modules for particular classes of blocks. It is still one of the major unsolved problems in the representation theory of general finite groups.
In the case of a block with abelian defect group, Alperin's Weight Conjecture simply claims that B and its Brauer correspondent b have the same number of simple modules (up to isomorphism). For this case, Broué formulated in 1989 a much more precise conjecture on the relationship between the two blocks. His Abelian Defect Group Conjecture claims that the derived categories $D^b(\mathrm{mod}-B)$ and $D^b(\mathrm{mod}-b)$ of the module categories of B and b are equivalent as triangulated categories. Since the number of simple B-modules is the rank of the Grothendieck group of $D^b(\mathrm{mod}-B)$, and hence an invariant of the derived category, Broué's conjecture implies that of Alperin for blocks with abelian defect group, but also says far more.
Character-theoretic and other circumstantial evidence for the Abelian Defect Group Conjecture accumulated rapidly, but verification of the conjecture for particular groups proved very difficult except in very simple cases. However, in the last two years there has been rapid progress in verifying special cases of the conjecture.
In my lecture, I will survey current knowledge on the Abelian Defect Group Conjecture, its consequences, and related questions, and describe some of the more recent work on special cases of the conjecture.
Reference: M. Broué, Isométries parfaites, types de blocs, catégories dérivées, *Astérisque* **181–182** (1990), 61–92.