# ICM 1998, Berlin, Aug. 18–27

## Abstracts of Plenary and Invited Lectures

### Section:

## 3. Number Theory and Arithmetic Algebraic Geometry

1991 MS Classification: 11 J 13, 11 J 68, 11 J 61, 11 D 61, 11 D 72
Schlickewei, Hans Peter, Philipps-Universität Marburg, Germany
**The Absolute Subspace Theorem and Applications**

1. The Subspace Theorem
The Subspace Theorem in Diophantine Approximation by W. M. Schmidt (1972) reads as follows:
*Let* $L_1, \dots, L_n$ *be linearly independent linear forms,* $L_i(\boldsymbol{X}) = \lambda_{i1}X_1 + \cdots + \lambda_{in}X_n$ *with algebraic coefficients* $\lambda_{ij}$*. Suppose* $\varepsilon > 0$*. Consider the inequality*

$$|L_1(\boldsymbol{x}) \dots L_n(\boldsymbol{x})| < |\boldsymbol{x}|^{-\varepsilon} \tag{1}$$

*in* $\boldsymbol{x} \in \mathbb{Z}^n$*, where* $|\boldsymbol{x}| = (x_1^2 + \cdots + x_n^2)^{1/2}$*. Then there exist finitely many proper linear subspaces* $T_1, \dots, T_t$ *of* $\mathbb{Q}^n$ *such that the set of solutions of* (1) *is contained in the union*

$$T_1 \cup \ldots \cup T_t. \tag{2}$$

Later on the Subspace Theorem has been extended to include a finite set $S$ of $p$-adic valuations and also to deal with the situation where the variables are taken from the ring of integers of a number field $K$. It is in fact this generalization that is most interesting for applications.
In the last decade quite some activity was going on to give uniform upper bounds for the *number* $t$ of subspaces $T_1, \dots T_t$ needed in (2), to cover the set of solutions of (1).
We will discuss in detail the most recent result in that context due to Evertse and Schlickewei, the *Quantitative Absolute Subspace Theorem* (to appear). Here the *variables are allowed to vary inside* $\overline{\mathbb{Q}}^n$ (the field of all algebraic numbers) *instead of* $\mathbb{Q}^n$ *or* $K^n$*.* Roughly speaking, if we disregard the "very small" solutions of (1), we obtain in this general situation an *upper bound for the number of subspaces in* (2) *that depends only upon the dimension* $n$ *and upon* $\varepsilon$*.*
2. Applications
We will give several applications to Diophantine equations.
Let us mention here one of them: Let $\{u_n\}_{n\in\mathbb{Z}}$ be a linear recurrence sequence of order $k$ with elements in a field of characteristic 0, i. e., a sequence satisfying a recurrence relation

$$u_{n+k} = a_{k-1}u_{n+k-1} + \cdots + a_0 u_n \quad (n \in \mathbb{Z}).$$

A classical conjecture has been as follows:
*If the sequence* $\{u_n\}$ *is "nondegenerate" then the number of solutions* $n \in \mathbb{Z}$ *of the equation*

$$u_n = 0$$

*is bounded in terms of the order* $k$ *of the sequence only.*
This conjecture now is a Theorem. It has been settled in the case when $\{u_n\}$ has only simple characteristic roots by Evertse, Schlickewei and W. M. Schmidt (to appear) and shortly afterwards by W. M. Schmidt (to appear) in general.