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Section:

3. Number Theory and Arithmetic Algebraic Geometry

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p-adic Hodge Theory in the semi-stable reduction case

I will give a brief survey of recent developments of p-adic Hodge theory in the semi-stable reduction case.
For a proper smooth algebraic variety X over the complex number field, Hodge theory relates the singular cohomology $H^*(X^{\mathrm{an}}, \mathbb{Q})$ of the analytic space X^{an} associated to X with the de Rham cohomology $H^*_{\mathrm{dR}}(X/\mathbb{C})$. p-adic Hodge theory is its analogue for a proper smooth algebraic variety X over a p-adic field (more precisely, a complete discrete valuation field of mixed characteristic $(0, p)$ with perfect residue field, e. g. a finite extension of $\mathbb{Q}_p$). The p-adic étale cohomology $H^*_{\text{ét}}(X_{\overline{K}}, \mathbb{Q}_p)$, which is a $\mathbb{Q}_p$-vector space of finite dimension endowed with a continuous linear action of the absolute Galois group of K, plays the role of the singular cohomology. The de Rham cohomology $H^*_{\mathrm{dR}}(X/K)$ is still defined in the p-adic case and is a K-vector space of finite dimension.
The p-adic Hodge Theory dates back to the works of J. Tate and A. Grothendieck ([9], [8]) on an abelian variety with good reduction and the conjecture of Tate ([9]) (the Hodge-Tate conjecture=C_{HT}): an analogue of the Hodge decomposition. By establishing a functor connecting p-adic étale and de Rham cohomologies endowed with various additional structures (called a "mysterious functor" by A. Grothendieck) , J. -M. Fontaine (and U. Jannsen in the semi-stable case) formulated several refined conjectures: the de Rham conjecture=C_{dR}, the crystalline conjecture=C_{crys} and the semi-stable conjecture=C_{st} ([4], [5], [6]). These conjectures were studied by many mathematicians such as M. Raynaud, J. -M. Fontaine, G. Laffaille, W. Messing, S. Bloch, K. Kato, G. Faltings and O. Hyodo, and the conjectures C_{HT}, C_{dR} and C_{crys} were already solved without any restriction by G. Faltings in the end of 1980s. There were two approaches to the conjectures: the method of syntomic cohomology and p-adic vanishing cycles by Fontaine-Messing and Kato, and the method of almost étale extensions by Faltings. (Recently another method was found by W. Niziol). For C_{st}, we needed the assumption $\dim X < (p-1)/2$ (K. Kato by the first method) or that X has a model with at most a semi-stable curve singularity (G. Faltings by the second one) at that time.
In these four years, the restriction was removed first by myself by the Kato's method (for $p \geq 3$ in 1994 and for $p = 2$ in 1996) [10] and then by G.

Faltings by his method (1996) [7] together with its generalization to non-constant coefficients and the relative case. Also a beautiful theory for p-torsion cohomologies in the semi-stable case was established by C. Breuil (1996 [1]) by overcoming the difficulty in treating the cohomologies degree-wise in a previous work of Faltings. Thanks to the alteration of de Jong [3], we can now derive easily all the other conjectures from C_{st} and further I proved, by the method of [2], that the p-adic étale cohomology of a proper variety is always a potentially semi-stable representation even if it has a singularity. I will survey these developments in my talk.

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