ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

4. Algebraic Geometry

1991 MS Classification: 14J32, 14J28, 14J60, 81T30
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Geometry and String Duality

Superstring theory may be said to have six manifestations:

1. Type I open superstring
2. Type IIA superstring
3. Type IIB superstring
4. $E_8 \times E_8$ heterotic string
5. $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string
6. Eleven-dimensional supergravity (or "M-theory")

Of these, the first 5 theories naturally live in ten dimensions while the last one lives in eleven dimensions.

In order to describe the world in a smaller number of dimensions one compactifies space-time. That is, one replaces the ten-dimensional space-time of string theory by a product $X \times M$ where X is some (small) compact space of d-dimensions and M is a flat Minkowski space of $10-d$ dimensions. (In the case of M-theory we let X be $(d+1)$-dimensional.) One then imagines M to be the flat space-time we observe in the universe.

The notion of duality is that one may take two types of string theory from our list above and compactify them in two different ways and achieve isomorphic physics in the space M. As an example, one may take the type IIA superstring and compactify on some Calabi–Yau threefold X (and so $d=6$) to describe four-dimensional space-time. One may also take the type IIB superstring and compactify it on some other Calabi–Yau threefold Y and hope to produce exactly the same physics. Since the physics in M is determined by the geometry of X and Y, this statement must imply some relationship between the geometry of X and Y.

This example is the string duality version of "mirror symmetry". That is, X and Y are said to be a mirror pair of Calabi–Yau threefolds if and only if the type IIA string compactified on X is "isomorphic to" the type IIB string compactified on Y. Although this was not the original statement of mirror symmetry it is now regarded as its most succinct definition.

The key to understanding such duality statements is to analyze the relationship between the geometry of X and the physics in M. The first step is to consider the moduli space of a class of theories. That is, one enumerates the number of free parameters in the physics and tries to associate each one to some data for X. One of the keys to the rich structure of string duality is that one typically uses more data for X than one would normally consider from classical geometric constructions. For example, in the case of mirror symmetry, as well as the complex structure and the Kähler form on X, one is also forced to consider an extra parameter $B \in H^2(X, \mathbb{R}/\mathbb{Z})$. This allows the Kähler form to be "complexified".

When one has identified the parameters which make up the moduli space one can attempt to map the moduli space of theories to the moduli space of its dual partner. This introduces a highly non-trivial geometrical correspondence between two families of compactifications. In the case of mirror symmetry this acts roughly by exchanging the complex structure with the complexified Kähler form. Consideration of the details of this map led to the celebrated enumeration of rational curves on X by Candelas et al.

Clearly we may analyze many more "mirror-symmetry-like" dualities other than the type IIA/IIB duality above. Indeed, the other dualities one would select from our list above should be expected to yield potentially more interesting correspondences since one is typically identifying more diverse objects.

One case which has been studied to a large extent in the past few years is that of heterotic/type IIA duality (particularly in the case of the $E_8 \times E_8$

heterotic string). That is, a type IIA string on X is dual to a $E_8 \times E_8$ heterotic string on Z. In this case, the data on the heterotic side must include the choice of a vector bundle on Z whose structure group is a subgroup of $E_8 \times E_8$ (or some sheaf-theoretic generalization of this).

In the case that X is a Calabi–Yau threefold, Z will be a product of a K3 surface and an elliptic curve. This duality therefore suggests some mapping from the moduli space of Calabi–Yau threefolds to the moduli space of K3 surfaces and elliptic curves together with some bundle data.

In this case, the elliptic curve part of the moduli space is fairly well-understood but that part associated to the K3 surface is somewhat more troublesome. The K3 part can be understood well in one special limit of the moduli space where X undergoes a rather drastic degeneration. Here the three-dimensional variation of Hodge structure on X decomposes as a variation of Hodge structure of a two-dimensional object and the variation of Hodge structure of a one-dimensional object. Here, the two-dimensional object can be identified as the required K3 surface and the one-dimensional object is a spectral curve specifying the vector bundle.