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Abstracts of Plenary and Invited Lectures

Section:

4. Algebraic Geometry

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Mirror symmetry and toric geometry

A *toric variety* of dimension n over $\mathbf{C}$ is a normal algebraic variety V containing $(\mathbf{C}\setminus\{0\})^n$ as a Zariski open subset in such a way that the algebraic group action of $(\mathbf{C}\setminus\{0\})^n$ on itself extends to a regular action on V. A projective toric variety V is called a *Gorenstein toric Fano variety* if V has at worst Gorenstein singularities and the anticanonical sheaf on V is ample. A convex n-dimensional polyhedron $\Delta \subset \mathbf{R}^n$ is called a *reflexive polyhedron* if the following conditions are satisfied: 1) all vertices of Δ belong to $\mathbf{Z}^n$; 2) the point $(0,\dots,0)$ belongs to the interior of Δ; 3) all vertices of the polar polyhedron

$$\Delta^* := \{(x_1,\dots,x_n) \in \mathbf{R}^n : \sum_{i=1}^n x_i y_i \geq -1 \;\; \forall (y_1,\dots,y_n) \in \Delta\}$$

belong to $\mathbf{Z}^n$. There exists a 1-to-1 correspondence between the isomorphism classes of n-dimensional Gorenstein toric Fano varieties V and n-dimensional reflexive polyhedra $\Delta \subset \mathbf{R}^n$ considered up to a linear transformation from $GL(n,\mathbf{Z})$. If Δ is a reflexive polyhedron, then the polar polyhedron Δ^* is again reflexive. The duality between reflexive polyhedra $\Delta \leftrightarrow \Delta^*$ induces a duality $V \leftrightarrow V^*$ between the corresponding Gorenstein toric Fano varieties. It turns out that Calabi-Yau hypersurfaces X in V are *mirror symmetric* to Calabi-Yau hypersurfaces X^* in V^*. Recently this observation became a powerful tool in the investigation of the mirror symmetry phenomenon discovered by physicists.

Reference: V. V. Batyrev, Dual polyhedra and mirror symmetry for Calabi-Yau hypersurfaces in toric varieties, *J. Algebraic Geometry* **3** (1994)