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Abstracts of Plenary and Invited Lectures

Section:

4. Algebraic Geometry

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Barsotti-Tate groups and crystals

Let G and H be Barsotti-Tate groups (also called p-divisible groups) over a discrete valuation ring R with field of fractions K. In 1966 J. Tate proved that, if the characteristic of K is zero, then any homomorphism between the generic fibres $G \otimes K \to H \otimes K$ extends to a homomorphism $G \to H$ over R. In the introduction of exposé IX in SGA 7 (by A. Grothendieck, M. Raynaud and D. Rim) it was mentioned as a problem whether the same holds when the characteristic of K is p.
As a first step in the solution to this problem, A. Grothendieck, W. Messing, P. Berthelot and others developed a crystalline Dieudonné module theory. This theory classifies Barsotti-Tate groups over a scheme of characteristic p in terms of Dieudonné crystals. The final step is to solve the resulting extension problem for morphisms of Dieudonné crystals and more generally for F-crystals (to appear in Inventiones Mathematicae).
In the talk we will indicate what kind of algebraic objects come up in the study of these Dieudonné crystals and F-crystals. The proof of the extension result involves a comparison of various types of convergence conditions on formal power series over rings of Witt-vectors. We will try to explain this.
Consider the following statement: Let F be a field finitely generated over its prime field $\mathbf{F}_p$, let X and Y be abelian varieties over F. Then the natural map

$$\mathrm{Hom}(X, Y) \otimes_{\mathbf{Z}} \mathbf{Z}_p \longrightarrow \mathrm{Hom}(X[p^\infty], Y[p^\infty])$$

is an isomorphism. When F is finite, this was proved by J. Tate (1966). For general F this result can be derived from the extension theorem on homomorphisms of p-divisible groups. This derivation is completely similar to J. Zarhin's proof of the ℓ-adic case. (It is amusing to note that G. Faltings proof of the same statement in the number field case used the extension theorem on homomorphisms for p-divisible groups proved by Tate.)
Finally, we will mention a conjecture on semi-stable reduction for so-called overconvergent F-crystals; such a conjecture occurs in work of R. Crew, N. Tsuzuki and others. The conjecture is a p-adic analogue of the strong results (quasi-unipotency of monodromy and the nilpotent orbit theorem) for variations of Hodge structure. It is an optimistic conjecture which implies among other things that certain rigid cohomology groups are finite dimensional, thereby tying in to recent results in this direction (by P. Berthelot, G. Christol, Z. Mebkhout and others).