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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

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Hard Balls Gas and Alexandrov Spaces of Curvature Bounded Above

Consider the hard balls model of statistical physics, that is a system of several hard balls moving freely and colliding elastically. One asks whether the number of collisions that may occur in this system can be estimated from above by a bound depending only on the number of balls and their masses? If we consider the balls moving in unbounded Euclidean space, we count the total number of collisions in infinite time. For a system of balls in a box, we mean the number of collisions in unit time (for a fixed value of kinetic energy). A "physical" version of these problems goes back to Boltzman, while their first mathematical formulation is probably due to Ya. Sinai.

In early 60th V. Arnold "speculated that such systems can be considered as the limit case of geodesic flows on negatively curved manifolds (the curvature being concentrated on the collisions hypersurface)" (quoted from Arnold's lecture at the meeting in the Fields institute dedicated to his 60th birthday). Indeed, this is nowadays well known (due to the works of Sinai, Bunimovich, Chernov, Katok, Strelcyn, Szasz, Simanyi and many others) that a large portion of the results in the smooth theory of (semi-)hyperbolic systems can be generalized (with appropriate modifications) for (semi-)dispersing billiards. In spite of this, the construction suggested by Arnold has never been used. It also caused several serious objections; in particular, A. Katok pointed out that such approximations by geodesic flows on manifolds necessarily produce geodesics bending around collision hypersurfaces and therefore having no analogs in the billiard system.

In the talk we will discuss elementary constructions representing billiard trajectories as geodesics in appropriate spaces. Unfortunately, these spaces are not even topological manifolds: they are lengths spaces of curvature bounded above in the sense of A. D. Alexandrov. Nevertheless, applying these constructions transforms a certain type of problems about billiards into purely geometric statements; and although a problem may seem rather difficult in its billiard clothing, its geometric counterpart may turn out to be relatively easy by the standards of the modern metric geometry. In particular, this approach allowed to solve the problems formulated at the beginning of the abstract by means of applying rather standard arguments in the theory of length spaces of curvature bounded above. We will also discuss new geometric problems arising in connection with these constructions.