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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

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Positive line bundles in symplectic geometry

In this lecture we will describe how certain familiar techniques in complex geometry can be applied to symplectic manifolds. The general setting is that we fix an almost-complex structure on a compact symplectic manifold (M, ω) and a unitary connection on a complex line bundle L over M whose curvature is $-i\omega$. In the classical case of a complex Kahler manifold V this would be a positive line bundle, and one knows that a high tensor power $L^{\otimes k}$ has many holomorphic sections. More precisely, these holomorphic sections yield a projective embedding of V, and generic projections give rational maps from V to other complex projective spaces. In the general case one studies "approximately" holomorphic sections of $L^{\otimes k}$, and applies these to produce objects with the same qualitative properties as the paradigms in complex geometry. The success of this technique hinges on certain refined transversality theorems, with estimates, to achieve uniform control with respect to the large parameter k. Results proved using this technique include existence theorems for symplectic submanifolds and Lefschetz fibrations . Combined with work of Gompf, we obtain, in principle, a complete topological characterisation of symplectic 4-manifolds.

These methods can also be applied to Lagrangian submanifolds and symplectomorphisms. Key roles are played by the generalised Dehn twists about Lagrangian spheres, which enter as the monodromy of a Lefschetz fibration around a single critical value. In the last part of the lecture we will describe some connections with recent work of P. Seidel , which involves the symplectic version of the Floer homology theory. This Floer theory, which has been developed by Fukaya and others, has an intricate algebraic structure—it turns the set of isotopy classes of Lagrangian submanifolds into a category—about which relatively little is known.