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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

1991 MS Classification: 53B50, 32G34, 35Q15, 35Q53, 52B

Dubrovin, Boris, SISSA, Trieste, Italy, and Steklov Math. Institute, Moscow, Russia

Geometry and analytic theory of Frobenius manifolds

Frobenius manifolds were invented as a coordinate-free form of the following *WDVV equations of associativity*: to find a quasihomogeneous function $F(t)$ of the variables $(t^1, \ldots, t^n) = t$ and a constant nondegenerate symmetric matrix $\left(\eta^{\alpha\beta}\right)$ such that the following combinations of the triple derivatives of $F(t)$

$$c_{\alpha\beta}^{\gamma}(t) := \sum_{\epsilon=1}^{n} \eta^{\gamma\epsilon} \frac{\partial^3 F(t)}{\partial t^\epsilon \partial t^\alpha \partial t^\beta}$$

for any t are structure constants of an associative algebra

$$A_t = \mathrm{span}(e_1, \ldots, e_n),\ e_\alpha \cdot e_\beta = \sum_{\gamma=1}^{n} c_{\alpha\beta}^{\gamma}(t) e_\gamma,\ \alpha,\, \beta = 1, \ldots, n$$

with the unity e_1. These equations were discovered by physicists E. Witten, R. Dijkgraaf, E. Verlinde and H. Verlinde in the beginning of '90s.

Main mathematical applications of Frobenius manifolds are in the theory of Gromov - Witten invariants of smooth algebraic varieties, in singularity theory, in differential geometry of the orbit spaces of reflection groups and their extensions, in the hamiltonian theory of integrable hierarchies. The theory of Frobenius manifolds establishes remarkable relationships between these, sometimes rather distant, mathematical theories.

Frobenius manifolds have amazingly rich geometrical structure. This allows to classify *semisimple* Frobenius manifolds where the algebra A_t is assumed to be semisimple for generic values of t. The moduli of n-dimensional semisimple Frobenius manifolds are the monodromy data of the linear differential operator with rational coefficients of the form

$$L = \frac{d}{dz} - U - \frac{1}{z} V$$

where U, V are z-independent $n \times n$ diagonal and antisymmetric matrices respectively. The Frobenius manifold structure is realized on the space of isomonodromy deformations of the operator L.

The basic ideas of this classification will be explained in the talk. It gives rise to new problems of the theory of isomonodromy deformations, reflection groups, and integrable hierarchies. Some of them will also be discussed in the talk.

References [1] B. Dubrovin, Integrable systems in topological field theory, Nucl. Phys. **B 379** (1992) 627 - 689. [2] B. Dubrovin, Geometry of 2D topological field theories, In: *Integrable Systems and Quantum Groups*, Eds. M. Francaviglia, S. Greco, Springer Lect. Notes in Math. **1620** (1996) 120 - 348. [3] B. Dubrovin, Painlevé transcendents in two-dimensional topological feild theory, math. AG/9803107, to appear in Proceedings of the school "The Painlevé property, one century later", Cargèse, 1996.