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Section:

5. Differential Geometry and Global Analysis

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Curvature-decreasing maps are volume-decreasing

The "Minimal Volume" of a differentiable manifold was introduced by M. Gromov in order to generalize (in any dimension) the Gauss-Bonnet-Chern-Weil formulas for characteristic classes and the induced estimates on volume and curvature. Giving a lower bound of the minimal volume in terms of the simplicial volume, M. Gromov obtained such a generalization. He thus conjectured that this inequality may be sharpened, i. e. that the minimal volume of a hyperbolic manifold is achieved by the hyperbolic metric.
We proved this conjecture by settling a real analogue of the Schwarz's lemma : if X, Y are two manifolds such that the curvature of X is negative and not greater than the one of Y, then any homotopy class of maps from Y to X contains a map which contracts volumes. We give a construction of this map which, under the assumptions of Mostow's rigidity theorems, turns to be an isometry, providing a unified proof of these rigidity theorems. It moreover proves that the set of all Einstein metrics, on any compact 4-dimensional hyperbolic manifold, reduces to a single point.
The entropy of a riemannian manifold is the limit of $\frac{1}{R}\,Log(Vol\,B(x,R))$, for big balls $B(x,R)$ in the riemannian universal covering of the manifold. The real Schwarz's lemma also gives a sharp inequality between the entropies of Y and X (provided that X is a compact locally symmetric negatively curved manifold and that there exists a map of non zero degree from Y to X, this inequality works for any compact riemannian manifold Y). This answers conjectures of A. Katok and M. Gromov about the minimal entropy of a manifold.
As this inequality is an equality iff Y is isometric to X, and as the volume and the entropy of a riemannian manifold are dynamical invariants, it implies that Y and X have conjugate geodesic flows iff they are isometric.
This also ends the proof of the Lichnerowicz's conjecture : any negatively curved compact locally harmonic manifold is a quotient of a (noncompact) rank-one-symmetric space.
We recently generalized this method to cases where $dim(X) \neq dim(Y)$ and where X is no longer assumed to be compact nor regular; we use it in order to obtain rigidity results for representations of discrete groups into SO(N,1)

and SU(N,1). The real Schwarz's lemma is also used in order to construct diffeomorphisms between negatively curved manifolds and hyperbolic ones (assuming almost maximal volume or pinched curvature).