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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

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Huisken, Gerhard, Universität Tübingen, Germany

Evolving hypersurfaces by their curvature

Let $F_0 : M^n \to (N^{n+1}, g)$ be a smooth immersion of a hypersurface in a Riemannian manifold (N^{n+1}, g). We study one–parameter families $F : M^n \times [0,T] \to (N^{n+1}, g)$ of hypersurfaces $M_t^n = F(\cdot, t)(M^n)$ satisfying a parabolic initial value problem

$$\begin{aligned} \frac{d}{dt}F(p,t) &= f\,\nu\,(p,t), \qquad\qquad p \in M^n, \quad t \in [0,T], \\ F(p,0) &= F_0(p), \end{aligned}$$

where ν is a choice of unit normal and f is some smooth function depending on the principal curvatures $\{\lambda_i\}_{1 \le i \le n}$ of the surface at the point $F(p,t)$.
Typical examples are the mean curvature flow $(f = H = \lambda_1 + \cdots + \lambda_n)$, the inverse mean curvature flow $(f = -1/H)$, the Gauss–curvature flow $(f = G = \lambda_1 \cdots \lambda_n)$, the affine curvature flow $(f = G^{1/(n+2)})$ and the harmonic mean curvature flow $\big(f = 1/(\lambda_1^{-1} + \cdots \lambda_n^{-1})\big)$. Based on the concept of a heat equation, the analytical structure of this system of evolution equations is closely related to the harmonic map heatflow and the Ricci flow of Riemannian metrics.
The last decade has seen major advances in the understanding of local and global properties of these parabolic deformations. The close interplay between analytical properties of the flow equation and geometrical assumptions on the underlying manifold has provided an elegant new approach to many classical results, e. g. to closed geodesics by curve–shortening, to sphere theorems by harmonic mean curvature flow and to the affine isoperimetric inequality via affine mean curvature flow.
Classical mean curvature flow $(f = H)$ is the L^2–gradient flow for the area functional and can be considered as the parabolic analogue of the minimal surface equation. For this flow some aspects of short–term and long–term behaviour are well understood, and new insight into the structure of singularities was obtained in recent years. In certain cases it seems now possible to attack the problem of extending the flow in a controlled way beyond singularities. In a weak setting mean curvature flow plays an important role as a model problem for the theory of interfaces and phase transitions.

In a recent development the inverse mean curvature flow ($f = -1/H$) has been used in a generalised setting to give a proof of the Riemannian Penrose inequality in General Relativity, providing an optimal lower bound for the total mass of an isolated gravitating system modelled by an asymptotically flat three manifold of non–negative scalar curvature, in terms of the size of a contained black hole modelled by an outermost minimal surface (joint work of G. Huisken and T. Ilmanen, see *IMRN No.* **20** (1997), 1045–1058). The lecture describes typical phenomena encountered in the geometric hypersurface evolutions above and explains some main techniques and results.