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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

1991 MS Classification:

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Geometry on the Group of Hamiltonian Diffeomorphisms

The group of Hamiltonian diffeomorphisms $\mathrm{Ham}(M,\Omega)$ of a symplectic manifold (M,Ω) plays a fundamental role both in geometry and classical mechanics. For a geometer, at least under some assumptions on the manifold M, this is just the connected component of the identity in the group of all isometries of the symplectic structure Ω. From the mechanical point of view, $\mathrm{Ham}(M,\Omega)$ is the group of all admissible motions. It was discovered by H. Hofer (1990) that the group of Hamiltonian diffeomorphisms carries a natural Finsler metric with non-degenerate distance function. Intuitively speaking, the distance between a given Hamiltonian diffeomorphism f and the identity transformation is equal to the minimal amount of energy required in order to generate f. This geometry has been intensively studied for the past 8 years in the framework of modern symplectic topology. It serves as a source of refreshing problems, and gives rise to new methods and notions. Also, it opens up the intriguing prospect of being able to use an alternative geometric intuition in Hamiltonian dynamics. In the talk I am going to focus on the following aspects of these developments.

- A paradox: locally the space $\mathrm{Ham}(M,\Omega)$ looks like a flat one, while globally that is not the case.

- A conjecture: the diameter of $\mathrm{Ham}(M,\Omega)$ is infinite. I shall discuss some existing methods, including a recent proof of this conjecture for $M = S^2$.

- Length spectrum and a symplectic version of Yang-Mills theory.

- Asymptotic geometric invariants.

- Applications to dynamics and ergodic theory.