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Abstracts of Plenary and Invited Lectures

Section:

5. Differential Geometry and Global Analysis

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Quantum cohomology and its applications

In a few years, quantum cohomology has grown from an infant in physics to young adult in mathematics with relations to different fields such as symplectic topology, algebraic geometry, quantum and string theory, integrable systems and gauge theory. The development in last few years has been explosive. Now, the foundations have been systematically studied and the ground is secure. As a young adult, quantum cohomology is looking forward to a steady and productive life ahead. Therefore, now is a particularly good time to assess the developments of last few years and make some proposals for the future. Quantum cohomology is such a diverse field that it is impossible to make a complete survey in 45 minutes. I will make no attempt to do so. I will refer B. Dubrovin's lecture in this congress for the algebraic side of the story.
The devoplement of quantum cohomology has roughly two distint periods: an early pioneer period and more recent period of technical sophistication. The main focus of early period was to establish a mathematical foundation for quantum cohomology. It was accomplished by author and G. Tian for semi-positive symplectic manifolds, a class which contains the most of interesting examples for applications such as Fano and Calabi-Yau manifolds. Another equally important activity of this period was the exploration of relation of quantum cohomology with other fields, like symplectic topology, string theory, gauge theory and birational geometry. These explorations lay down a foundation for future applications. I will give a rather detailed survey of this period.
The recent period focused on removing the semi-positivity condition: this was done recently by an array of authors. The main characteristic of this period is increasingly technical sophistication. In my view, the development of these new techniques is probably more important than the goal of removing semi-positivity condition itself. Therefore, I will mostly survey the main techniques in this period.
The most sucessful application of quantum cohomology right now is in enumerative geometry. Due to my limited expertise on this subject, I will leave it to the experts to give a proper survey. Instead, I will focus on other aspect of applications, for examples, functorial properties of quantum cohomology, application to symplectic topology and birational geometry. I believe that there is a great potential lying on these applications.