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6. Topology

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Knot Surgery on 4-Manifolds

In this talk we shall discuss geometric operations on a given smooth 4-manifold which preserve the underlying topological structure and alter its smooth structure. In particular, we shall consider the following construction: Let X be a simply connected smooth 4-manifold which contains a smoothly embedded torus T of self-intersection 0. Given a knot K in S^3, we replace a tubular neighborhood of K with $S^1 \times (S^3 \setminus K)$ to obtain the *knot surgery manifold* X_K.

More formally, this procedure is accomplished by performing 0-framed surgery on K to obtain the 3-manifold M_K. The meridian m of K can be viewed as a circle in M_K; so in $S^1 \times M_K$ we have the smooth torus $T_m = S^1 \times m$ of self-intersection 0. Since a neighborhood of m has a canonical framing in M_K, a neighborhood of the torus T_m in $S^1 \times M_K$ has a canonical identification with $T_m \times D^2$. The knot surgery manifold X_K is given by the fiber sum

$$X_K = X \#_{T=T_m} S^1 \times M_K = (X \setminus T \times D^2) \cup (S^1 \times M_K \setminus T_m \times D^2)$$

where the two pieces are glued together so as to preserve the homology class $[\mathrm{pt} \times \partial D^2]$. This latter condition does not, in general, completely determine the diffeomorphism type of X_K, and X_K is taken to be any manifold constructed in this fashion.

We shall discuss consequences of this knot surgery construction. For example, under relatively mild hypotheses on (X, T), X_K will be homeomorphic to X, and its Seiberg-Witten invariant (viewed as a Laurent polynomial) will be $\mathcal{SW}_{X_K} = \mathcal{SW}_X \cdot \Delta_K(t)$, where $\Delta_K(t)$ is the symmetrized Alexander polynomial of K in the variable $t = \exp(2[T_m])$. As a consequence, given (X, T) satisfying the hypotheses, this produces infinitely many nondiffeomorphic 4-manifolds, all homeomorphic to X. In particular, this works for $(K3, \text{elliptic fiber})$. If the knot K is fibered, and X is symplectic with T a symplectically embedded torus, then the knot manifold X_K is also symplectic. However, if the Alexander polynomial of K is not monic, the knot manifold X_K supports no symplectic structure.

We shall also use a similar technique to construct exotic embedded surfaces in 4-manifolds, and discuss what is known about geography of smooth 4-manifolds.

Reference: R. Fintushel and R. Stern, *Knots, links, and* 4-*manifolds*, to appear in Invent. Math.