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Abstracts of Plenary and Invited Lectures

Section:

7. Lie Groups and Lie Algebras

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Towards a stable trace formula

Suppose that G is a connected, reductive group over a number field F. For example, we could take F to be $\mathbb{Q}$, and G to be a representative from one of the four infinite families $GL(n)$, $SO(2n+1)$, $Sp(2n)$ and $SO(2n)$. The trace formula for G is an identity that relates geometric objects with dual spectral objects. The geometric objects are attached to conjugacy classes in $G(F)$, and are closely related to harmonic analysis. The spectral objects are attached to automorphic representations, and are really the ultimate objects of study. They carry fundamental arithmetic information, that is believed to hold the answers to some of the deepest questions in number theory.

It is hard to extract arithmetic information from the trace formula by studying it in isolation. There are simply too many geometric terms. One should try instead to compare it with trace formulas for other groups. Such a comparison could lead to fundamental reciprocity laws between automorphic spectra on different groups. In the case of the classical groups $SO(2n+1)$, $Sp(2n)$ and $SO(2n)$, it would provide the means for attacking a basic classification problem, that of reducing the representation theory of these groups to the much better understood theory of $GL(n)$.

The groups to be compared with a given G are the *endoscopic* groups for G. They form a family of quasisplit groups $\{G'\}$ that have many conjugacy classes in common with G. It is actually only the stable conjugacy classes –equivalence classes in $G(F)$ defined by conjugacy over an algebraic closure $\bar{F}$ — that can be transferred from G to other groups G'. On the other hand, the geometric terms in the trace formula are not stable distributions. They depend on more than just the averages of a test function over stable conjugacy classes. The basic problem can then be formulated as that of stabilizing the trace formula. It entails the construction of a stable trace formula –a refined identity whose individual terms are stable distributions. It also includes the further analysis required to establish identities between terms in the original trace formula and their stable counterparts on endoscopic groups.

The lecture will be a general discussion of questions relating to the construction and analysis of a stable trace formula. I can say nothing about the fundamental lemma, which is one of the key problems to be resolved. However, many other analytic and combinatorial problems have been solved.

I shall describe the relationships of these problems to the local and global terms in the trace formula.