ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

7. Lie Groups and Lie Algebras

1991 MS Classification: 20E06, 20E08, 20E32, 20F32, 22D40, 22E40
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Products of trees, lattices and simple groups

The group of automorphisms of a locally finite tree, denoted $\mathrm{Aut}\, T$, is a locally compact group which exhibits behaviour analogous to that of a rank one simple Lie group. This analogy has motivated many recent works, in particular the study of lattices in $\mathrm{Aut}\, T$ by Bass, Kulkarni, Lubotzky and others. Recall that in the case of semisimple Lie groups, irreducible lattices in higher rank groups have a very rich structure theory and one encounters many deep and interesting phenomena such as (super)rigidity and arithmeticity. Motivated by this we study in several joint works with Marc Burger and with Robert J. Zimmer cocompact lattices in the group of automorphisms of a product of trees or rather in groups of the form $\mathrm{Aut}\, T_1 \times \mathrm{Aut}\, T_2$ where each T_i, $i = 1, 2$ is a (bi-)regular tree. The results obtained concerning the structure of lattices in $\mathrm{Aut}\, T_1 \times \mathrm{Aut}\, T_2$ enable us to show the existence and construct the first examples of finitely presented torsion free simple groups, see [1].

A lattice $\Gamma < \mathrm{Aut}\, T_1 \times \mathrm{Aut}\, T_2$ is reducible if its projections on each factor $\mathrm{Aut}\, T_i$ are discrete. Denoting the closures of the projections of Γ by $H_i = \overline{\mathrm{pr}_i(\Gamma)}$, $i = 1, 2$, we observe that for cocompact lattices $H_i \neq \mathrm{Aut}\, T_i$. We show that imposing various "largeness" conditions on the groups H_i, $i = 1, 2$ allows one to obtain a rich structure theory for these lattices. A (closed) subgroup $H < \mathrm{Aut}\, T$ will be called *locally primitive* if for every vertex $x \in T$ its stabilizer in H acts as a primitive permutation group on the set of adjacent edges.

The following result may be viewed as an "arithmeticity theorem".

Theorem 1, [2]. *Let $\Gamma < Aut\, T_1 \times Aut\, T_2$ be an irreducible cocompact lattice. Let $H_i = \overline{pr_i(\Gamma)}$ be the closures of the projections, $i = 1, 2$. Assume that each $H_i < Aut\, T_i$ is locally primitive. Then one of the following possibilities holds:*
(1) Every linear image of Γ is finite.
(2) Γ has an infinite linear image over a field of characteristic 0. Then H_i is a p_i-adic analytic group for some prime p_i. The adjoint map yields a continuous surjection from $H_1 \times H_2$ onto a semisimple Lie group over some local fields and the image of Γ is an arithmetic lattice. Moreover the kernel of this homomorphism is a torsion free discrete subgroup of $H_1 \times H_2$.

(3) Γ has an infinite linear image over a field of positive characteristic p. Then there is a continuous map with unbounded image from $H_1 \times H_2$ into a simple Lie group over $F_p((t))$.

A related result is a geometric rigidity theorem asserting that if $\Gamma < \mathrm{Aut}\,(T_1) \times \mathrm{Aut}\,(T_2)$ is a cocompact lattice with locally primitive projections and Γ is isomorphic to a lattice $\Gamma' < \mathrm{Aut}\,(T_1') \times \mathrm{Aut}\,(T_2')$ then the isomorphism is induced by an isometry between $T_1 \times T_2$ and $T_1' \times T_2'$.
Further results, jointly with Marc Burger, study the normal subgroups structure of irreducible lattices in $\mathrm{Aut}\,T_1 \times \mathrm{Aut}\,T_2$. Again one needs to put conditions on the "size" of the projections of the lattice. For a locally compact group H let us denote by $H^{(\infty)}$ the intersection of all its cocompact closed normal subgroups. Assuming that the closures of the projections of an irreducible lattice act transitively on the boundary of the tree we have the following analog of Margulis' normal subgroup theorem:

Theorem 2, [1]. *Let $\Gamma < Aut\,T_1 \times Aut\,T_2$ be a cocompact lattice such that $H_i := \overline{pr_i(\Gamma)}$ acts transitively on ∂T_i and $H_i^{(\infty)}$ is of finite index in H_i, $i = 1, 2$. Then, any nontrivial normal subgroup of Γ is of finite index.*

Unlike finitely generated linear groups which are necessarily residually finite, lattices in $\mathrm{Aut}\,T_1 \times \mathrm{Aut}\,T_2$ may be non residually finite. We establish a criteria for ensuring that certain lattices are not residually finite. Using this criteria together with theorem 2 we show

Theorem 3, [1]. *For every pair (n, m) of sufficiently large even integers there exists a finite square complex $\mathcal{Y}_{n,m}$ whose universal covering is the product $\mathcal{T}_n \times \mathcal{T}_m$ of regular trees of respective degrees n and m and whose fundamental group $\Gamma_{n,m}$ (which is a cocompact lattice in $Aut\,\mathcal{T}_n \times Aut\,\mathcal{T}_m$) satisfies:*
(1) The group $\Gamma_{n,m}$ is simple, finitely presented and torsion free.
(2) $\Gamma_{n,m} \equiv F *_G F$ where F, G are finitely generated free groups, $[F : G] \in \{n, m\}$.
(3) If $\Gamma_{n,m}$ is isomorphic to $\Gamma_{k,l}$, then the corresponding complexes $\mathcal{Y}_{n,m}$ and $\mathcal{Y}_{k,l}$ are isomorphic, and $\{n, m\} = \{k, l\}$.
(4) The cohomological dimension of $\Gamma_{n,m}$ is 2.
(5) $\Gamma_{n,m}$ is an automatic group.

We remark that the question whether amalgams $F *_G L$ of free groups can be simple was raised by P. M. Neumann [4], [3].

References

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