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Abstracts of Plenary and Invited Lectures

Section:

8. Analysis

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Singularity and Regularity — Local and Global

For some years a central question in complex analysis in several variables was whether the Bergman projection (the orthogonal projection of square integrable functions onto holomorphic functions) preserves the class of functions smooth up to the boundary, for arbitrary smoothly bounded pseudoconvex domains. This is closely linked with the regularity of solutions of the $\bar{\partial}$–Neumann problem, a boundary value problem for a system of Laplace type partial differential equations in which the boundary conditions depend on the complex geometry of the domain's boundary. The question was finally settled in the negative, but by indirect reasoning which applied only to special examples. Despite a later simplification by Y. -T. Siu, certain more refined questions remain open.
Global regularity of the Bergman projection is related to other unresolved problems concerning the hypoellipticity and global regularity of subelliptic and more degenerate second order partial differential operators, in the real analytic, Gevrey, and smooth categories. The positive results for these problems are roughly parallel, both in scope and in the techniques from which they can be obtained. The negative analyses are more *ad hoc* though common themes do exist. Large gaps between the positive and negative results persist, except in the lowest-dimensional case.
Proofs of the positive results motivate the construction of a metric on the cotangent bundle which is somewhat related to the usual Carnot-Caratheodory metric on the base space, but differs in fundamental respects. Asymptotic properties of this metric correspond well with sharp hypoellipticity results, in cases where such results are known.
This talk will survey some aspects of these problems, emphasizing the negative results.

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