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## Abstracts of Plenary and Invited Lectures

### Section:

### 8. Analysis

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**On the Bilinear Hilbert Transform**

In 1964, Alberto Calderón defined a bilinear generalization of the Hilbert transform by

$$Hfg(x) := \text{p.v.} \int f(x+y)g(x-y)\,\frac{dy}{y}.$$

He raised the striking conjecture that this operator would map $L^2 \times L^2$ into $L^1$, in contrast to the well–known fact that the ordinary Hilbert transform does not preserve $L^1$. In joint work with Christoph Thiele, a range of inequalities for this transform have been established, including Calderón's conjecture. This talk will survey our argument.
Calderón's original motivation was the so–called commutator connected to the Cauchy integral on Lipschitz curves. The study of this integral lead to the development of many aspects of the modern Calderón–Zygmund Theory, but yielded no information about the bilinear Hilbert transform. Indeed, the fundamental obstacle was that in contrast to most operators of that theory, the bilinear Hilbert transform commutes with conjugation by exponentials in a natural way: Denoting $f_\lambda(x) = e^{i\lambda x} f(x)$, one sees immediately that $Hf_\lambda g_\lambda(x) = e^{2i\lambda x} Hfg(x)$. Thus, the transform displays a great deal of sensitivity to the frequency behaviors of the transformed functions.
There is one other operator which exhibits the same kind of behavior, and it appears in a celebrated theorem of Lennart Carleson on Fourier series. This Theorem, which ironically was established in 1965, asserts the pointwise convergence of the partial Fourier sums of a square integrable function on the unit circle. The operator that must be controlled to prove this result is the maximal partial Fourier sums of square–integrable function. It is again highly sensitive to the frequency behaviors of the function under consideration.
The proof which I developed with C. Thiele can be seen as an outgrowth and reëxamination of the deep and subtle techniques invented by L. Carleson in 1965. This talk will define the Hilbert transform, recall Carleson's Theorem, elaborate on it's relationship to the bilinear Hilbert transform, and describe elements of the proof of the Calderón conjecture, which is based upon combinatorial–geometric aspects of the time–frequency plane.