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8. Analysis

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Wave Equations with Low Regularity Coefficients

To a Riemannian manifold (M, g) is naturally associated the wave equation $\partial_t^2 u = \Delta_g u$, where Δ_g is the Laplace-Beltrami operator on M. For many applications it suffices to study this equation in local coordinates, in which it takes the form

$$\partial_t^2 u(t,x) \;=\; \frac{1}{\sqrt{g}} \sum_{i,j=1}^{n} \partial_{x_i}\big(\sqrt{g}\; g^{ij}(x)\; \partial_{x_j} u(t,x)\,\big)\,.$$

In case the metric coefficients $g^{ij}(x)$ are smooth functions of x, the local behavior of solutions is well understood through geometric optics and the Lax construction of the wave group for g. Such techniques, which lead to the use of Fourier integral operators, can be exploited to obtain space-time L^p norm estimates on solutions u which are important in the study of nonlinear versions of the wave equation. Examples include the Strichartz estimates, and the more recent null-form estimates of Klainerman-Machedon.

Much less is known in the case where the metric coefficients $g^{ij}(x)$ possess few derivatives with respect to x. For example, the following question is of interest: how many derivatives do the $g^{ij}(x)$ need in order to insure that, for example, the Strichartz estimates hold? We show that a sufficient condition is that the coefficients $R_{ijkl}(x)$ of the Riemann curvature tensor associated to g be bounded measurable functions; this is equivalent to uniform bounds on the sectional curvatures of (M, g). Counterexamples in joint work with C. Sogge show that L^q boundedness of the curvature is not sufficient if $q < \infty$.

The construction of the wave group which establishes this result is based upon a decomposition of phase space that has long been used to understand the behavior of oscillatory integral / Fourier integral operators on the L^p spaces. We show how this dyadic / parabolic decomposition, coupled with modified paraproduct techniques, leads to an appropriate geometric optics picture of solutions to wave equations for metrics with bounded curvature tensor. This construction is necessarily less precise than for smooth metrics, yet is sufficient to obtain basic estimates of interest for nonlinear wave equations.