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8. Analysis

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From finite- to infinite-dimensional phenomena in Geometric Functional Analysis, on local and asymptotic levels

We shall present several directions in which close connections can be found between the geometry and linear-metric structure of infinite-dimensional Banach spaces and finite-dimensional phenomena. The geometry of convex bodies in $I\!R^n$ has a fundamental influence on structural phenomena in random quotients of the corresponding normed spaces; and this in turn can be used in infinite-dimensional constructions with various non-local properties. Other directions use, to various extents, a logical connection between the finite-dimensional (or local) structure of a Banach space and its infinite-dimensional structure.

Pairs of finite-dimensional spaces with unsually large norms of isomorphisms were discovered by Gluskin in the early 80's (presented at the ICM '86); these were random quotients of ℓ_1^n of dimension $[n/2]$, say; and extensive investigations of this class of spaces by a group of researchers followed. We shall discuss analogous phenomenona for general normed spaces. Let $X = (I\!R^n, K)$ be an n-dimensional normed space with unit ball K. A normed space $E = (E, P_E(K))$, where E is a "random" $[n/2]$-dimensional subspace of $I\!R^n$ and $P_E(K)$ is the image of K under the projection onto E, turns out to have very few well-bounded operators, depending on the volume of K. Precise quantitative formulations are asymptotic, as $n \to \infty$. On the other hand, the volume of K also controls "closeness" of $P_E(K)$ to an ellipsoid. This indicates a type of curious dichotomy for general random spaces.

We shall show how the above phenomena can be used in the constructions of infinite-dimensional Banach spaces that do not admit continuous operators, or sequences of operators, with natural (algebraic) properties. In fact, such constructions require just a few canonical operations, starting from an arbitrary Banach space, non-isomorphic to ℓ_2. This in particular leads to characterizations of a Hilbert space.

Another point of view that we shall discuss depends on stabilization of the structure of a given Banach space X over infinite-dimensional subspaces. For example, if X has a basis, the asymptotic structure of X comprises all finite-dimensional subspaces spanned by blocks of the basis living sufficiently far along the basis. So called asymptotic-ℓ_p spaces, for which the asymptotic

structure is the simplest possible, generalize classical spaces ℓ_p, and they may still have an inherently different (and more interesting) geometry.
As discovered in a series of deep results presented at the ICM '94 in the talks by Gowers and by Odell and Schlumprecht, infinite-dimensional phenomena in general do not stabilize. Thus, a bridge between finite- and infinite-dimensional properties of Banach spaces is formed by localizing natural structural invariants to families of finite subsets of $\mathbb{N}$ with increasing complexity. As well, Gowers' Dichotomy Theorem may immediately reduce initially purely infinite-dimensional problems to local considerations.