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Abstracts of Plenary and Invited Lectures

Section:

9. Ordinary Differential Equations and Dynamical Systems

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Hyperbolicity, stability and the creation of homoclinic points

A homoclinic point is a point in the intersection of stable and unstable manifolds of the same hyperbolic fixed point (or periodic point). There are two main results. The first is a perturbation result (the connecting lemma) to create homoclinic points, whose orbits (homoclinic orbits) make complex dynamic behavior and hyperbolicity. As an application of this perturbation technique, we get, for stable vector fields, the separation of singularities from periodic points, which is the crucial step in the proof of the Stability Conjecture (raised by J. Palis and S. Smale in the late 1960's) for flows. This is the second result and, as consequence, it turns out that two concepts, hyperbolicity and stability are essentially equivalent to each other for C^1 dynamical systems of a compact manifold. Moreover we consider an application toward the solution of a conjecture by J. Palis saying that it is a generic property of diffeomorphisms on a compact manifold to have either simple dynamics or homoclinic orbits. This is a step to understand nonhyperbolic dynamics in line with Palis' program.