ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

9. Ordinary Differential Equations and Dynamical Systems

1991 MS Classification:
Grzegorz Świątek, The Pennsylvania State University, USA
Induced Hyperbolicity for One-Dimensional Maps

In the early 1980s M. Jakobson proved a theorem which asserted that in the logistic family $x \to ax(1-x)$, the mapping has a probabilistic absolutely continuous invariant measure for a set of parameters a with positive measure. The crucial step of the proof was the construction of an expanding map Φ, defined on the union of countably many intervals, so that on each connected component of its domain Φ was an iterate of the original quadratic map, and the range of this restriction was a fixed interval. Hence, from the original clearly non-expanding transformation, a uniformly expanding one was constructed by taking iterates in a piecewise fashion.
In the early 1990s J. -Ch. Yoccoz proved a theorem about local connectivity of Julia sets of some quadratic polynomials, including many Julia sets which contained the critical point. He also showed that for this class of polynomials certain combinatorial data, which for real maps reduces to the kneading sequence, determines that polynomial uniquely. The proof is based on the construction of partitions of the phase space which are not far from being Markov: most pieces are mapped in a univalent way onto other pieces, except for the pieces which contain the critical point. The partitions were subjected to a process of infinite inductive refinement under which the critical pieces shrank to the point.
It was then observed that a powerful tool for studying both unimodal maps of the interval and complex polynomials in the plane is obtained when these ideas are applied jointly. When appropriate iterations of the original polynomial are applied on pieces of Yoccoz partitions, the result is a map which is expanding on all pieces except for the one which contains the critical point. Such transformations are called *induced mappings.* When a sequence of increasingly refined partitions is considered, the critical branches disappear in the limit and an expanding map of Jakobson's type is obtained. The original construction of Jakobson required exclusion of unsuitable parameters without regard for their topological dynamics, although a set of positive measure was left at the end. The new approach based on Yoccoz' discovery works for all polynomials without neutral or attracting periodic orbits, with the only exception of a well defined class of infinitely tunable ones.

This, or a closely related approach, played the key role in the solution of several important problems. For real quadratic polynomials, and by extension for unimodal maps with non-degenerate critical point, the infinitely tunable case turned out to be quite manageable, due to the phenomenon of the so-called a priori bounds, discovered by D. Sullivan. The combination of inducing and a priori bounds was the basis of the result of G. Levin and S. Van Strien about local connectivity of Julia sets for all real unimodal polynomials. J. Graczyk and the author used the same ingredients, and another phenomenon realted to inducing and known as the *decay of geometry*, to prove that periodic windows are dense in the logistic family.
Finally, there are new results we wish to present which use inducing applied to complex polynomials. These results were obtained together with J. Graczyk and S. Smirnov and concern the behavior of the polynomials on the boundary of the Mandelbrot set, typical with respect to the harmonic measure. Among other properties, it turns out that such polynomials have Julia sets which are conformally removable and of Hausdorff dimension strictly less than 2. The first result extends a theorem by P. Jones and the second is complementary to a result by M. Shishikura which asserts that a complex polynomial in the residual subset of the boundary of the Mandelbrot set has Julia set with Hausdorff dimension equal to 2.