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10. Partial Differential Equations

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Bethuel, Fabrice, Université de Paris-Sud à Orsay, France

Recent results for Ginzburg-Landau models

These models were first introduced by V. Ginzburg and L. Landau around 1950 in order to describe superconductivity. Similar models appeared soon after for various phenomena: Bose condensation, superfluidity, non linear optics. A common property of these models is the major role of topological defects, termed in our context vortices.

In a joint book with H. Brezis and F. Helein, we considered a simple model situation, involving a bounded domain Ω in R^2, and maps v from Ω to R^2. The Ginzburg-Landau functional, then writes

$$E_\epsilon(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 + \frac{1}{4\epsilon^2}\int_\Omega (1-|v|^2)^2$$

Here ϵ is a parameter describing some characteristic lenght. We are interested in the study of stationary maps for that energy, when ϵ is small (and in the limit ϵ goes to zero). For such map the potential forces $|v|$ to be close to 1 and v will be almost S^1-valued. However at some point $|v|$ may have to vanish, introducing defects of topological nature, the vortices. An important issue is then to determine the nature and location of these vortices.

We will also discuss recent advances in more physical models like superconductivity, superfluidity, as well as for the dynamics: as previously the emphasis is on the behavior of the vortices. These results involve a very large number of contributions by many authors: Almeida, Colliander, Jerrard, Hardt, Lin, Mironescu, Rivière, Saut, Serfaty, Shafrir, Sigal and Struwe among others.