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Abstracts of Plenary and Invited Lectures

Section:

10. Partial Differential Equations

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On nonlinear dispersive equations

I shall present results obtained jointly with Carlos E. Kenig and Luis Vega concerning properties of solutions to nonlinear dispersive models. We consider equations of the form

$$\partial_t u = iP(\nabla_x)u + F(u, \partial_x u, \dots), \quad t \in \mathbb{R}, \quad x \in \mathbb{R}^n, \tag{1}$$

where the real symbol $P(i\vec{\xi})$ represents the dispersive relation and $F(\cdot)$ denotes the nonlinearity. These models which include the generalized Korteweg-de Vries equation ($n = 1,\ P(i\xi) = \xi^3,\ F = u^k \partial_x u,\ k = 1, 2...$), the generalized Benjamin-Ono equations, and systems related to the nonlinear Schrödinger equation, arise in several physical situations and have been shown to be relevant in different mathematical contexts. We are interested in the initial value problem (IVP) and the periodic boundary value problem (PBVP) associated to (1) and mainly study the following two related problems.
First, we deal with the problem of the optimal regularity of the datum $u(x, 0) = u_0(x)$ which guarantees that the corresponding IVP and PBVP are well-posed. The notion of well-posedness includes existence, uniqueness, continuous dependence of the solution upon the data u_0 in some function space $\mathcal{X}$, persistence property (i. e. the time evolution of the solution $u(t)$ describes a continuous curve in $\mathcal{X}$), and so the solutions generate a dynamical system in the function space $\mathcal{X}$. The regularity of the data is measured in the classical Sobolev spaces $H^s(\mathbb{R}^n) = (1 - \Delta)^{s/2} L^2(\mathbb{R}^n)$, and $\dot{H}^s(\mathbb{R}^n) = (-\Delta)^{s/2} L^2(\mathbb{R}^n)$ with $s \in \mathbb{R}$, in which solutions of the associated linear problem are given by the unitary group $\Gamma = \{e^{itP(\nabla_x)} \ :\ t \in \mathbb{R}\}$. We have developed new techniques, mainly coming from harmonic analysis, which provide a better understanding of the relationship between the dispersive (linear) part and the degree and the structure of the nonlinearity F. These include optimal estimates for smoothing effects and the maximal function associated to the group Γ. The methods allow us to establish local and global well-posedness results for low values of the index s and prove that in several cases these are the best possible.
The second problem addresses the existence of solutions to some nonlinear dispersive models of Schrödinger type. These include the Davey-Stewartson,

Ichimori and Zakharov-Schulman systems. In these systems the dispersion is represented by a non-degenerate second order operator

$$P = \partial^2_{x_1} + .. + \partial^2_{x_k} - \partial^2_{x_{k+1}} - ..\partial^2_{x_n},$$

(thus for $1 \geq k < n-1$ the operator P is not elliptic) and the nonlinearity F involved non-local operators of "orden one". In many cases the existence of solutions for the IVP associated to these systems cannot be obtained by standard methods such as : classical method, energy estimates, L^qL^p-estimates, etc., even assuming high regularity on the datum u_0. In this case we refine and extend arguments used in the previous case to answer the local well-posedness question for several forms of these systems.