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Inverse Boundary Value Problems for Partial Differential Equations

Inverse boundary problems are a class of problems in which one seeks to determine the internal properties of a medium by performing measurements along the boundary of the medium. These inverse problems arise in many important physical situations, ranging from geophysics to medical imaging to the non-destructive evaluation of materials.

The appropriate mathematical model of the physical situation is usually given by a partial differential equation (or a system of such equations) inside the medium. The boundary measurements are then encoded in a certain boundary map. The inverse boundary problem is to determine the coefficients of the partial differential equation inside the medium from knowledge of the boundary map.

In this talk we will survey part of the significant progress which has been made in the last twenty years in this area. Many of the advances have been a consequence of the construction of *complex geometrical optics* solutions for the class of partial differential equations under consideration.

The prototypical example of an inverse boundary problem is the inverse conductivity problem, also called electrical impedance tomography, first proposed by A. P. Calderón. Let Ω be a bounded domain with Lipschitz boundary. The electrical conductivity of Ω is represented by a bounded and positive function $\gamma(x)$. In the absence of sources or sinks of current the equation for a voltage potential u on Ω is given by

$$\operatorname{div}(\gamma \nabla u) = 0 \text{ in } \Omega\,. \tag{1}$$

In this case the boundary map is the voltage to current map; that is, the map assigns to a voltage potential on the boundary the corresponding induced current flux at the boundary. This boundary map is also called the Dirichlet to Neumann map, since the voltage potential is Dirichlet data associated to equation (1), and the induced current flux is Neumann data. The inverse problem in this case is to recover in Ω the function γ from the Dirichlet to Neumann map.

Many results on this particular inverse problem have been obtained using complex geometrical optics solutions to equation (1). We will also discuss other examples of inverse boundary problems, including examples associated to the Schrödinger equation in the presence of a magnetic field, Maxwell's equations, the Dirac equation, and the Lamé system of elasticity.