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Abstracts of Plenary and Invited Lectures

Section:

10. Partial Differential Equations

1991 MS Classification:

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Scattering theory: some old and new problems

Scattering theory is, roughly speaking, perturbation theory of self-adjoint operators on the (absolutely) continuous spectrum. It has its origin in mathematical problems of quantum mechanics and is intimately related to the theory of partial differential equations. This talk is devoted to a survey of different tools of this theory ranging from the abstract functional analysis to an advanced (and sometimes non-standard) calculus of pseudo-differential operators. Some recently solved problems, such as asymptotic completeness for the Schrödinger operator with long-range and multiparticle potentials, as well as open problems, are discussed. Special attention is paid to the properties of the scattering matrix which is the main observable of the theory.

Here are the detailed contents of the talk:

1. Two problems of scattering theory: eigenfunction expansions and large time asymptotics of unitary groups

2. Hamiltonians of quantum mechanics

3. Methods of scattering theory: “trace class“ and “smooth”. The absence of a bridge between these two methods. An open concrete problem

4. The multiparticle Schrödinger operator with short-range pair potentials
a) Setting up the problem
b) The Mourre estimate and the limiting absorption principle
c) The radiation conditions a priori estimate on the resolvent
d) The asymptotic completeness
e) An unsolved problem: the discrete version (Heisenberg model)

5. The multiparticle Schrödinger operator with long-range pair potentials
a) The asymptotic completeness
b) Some cases of the break-down of asymptotic completeness:
new channels of scattering

6. Scattering of waves on unbounded obstacles

7. The scattering matrix for the two-particle Schrödinger operator with a short-range potential. Its eigenvalues and eigenfunctions

8. The scattering matrix for the two-particle Schrödinger operator with a long-range potential