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11. Mathematical Physics

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Scaling algebras in local relativistic quantum physics

Relativistic quantum physics in a pseudo-Riemannian spacetime manifold $(\mathcal{M}, g)$ can conveniently be described by a C^*-dynamical system $(\mathcal{A}, \alpha)$, where $\mathcal{A}$ is a C^*-algebra, describing the physical observables in $\mathcal{M}$, and α a representation of the isometry group of $(\mathcal{M}, g)$ by automorphisms of $\mathcal{A}$. The Maxwell-Einstein principle of locality is implemented in this setting by exhibiting a distinguished pre-cosheaf (net) of subalgebras $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}$ which is labelled by the spacetime regions $\mathcal{O} \subset \mathcal{M}$.
In high energy physics the structure of the observables in very small spacetime regions $\mathcal{O}$ (at "small spacetime scales") is of great interest. It can be explored with the help of the new mathematical concept of *scaling algebra*. Given a dynamical system $(\mathcal{A}, \alpha)$, the elements of the associated scaling algebra $\underline{\mathcal{A}}$ consist of certain specific functions $\underline{A} : \mathbf{R}_+ \longmapsto \mathcal{A}$ which establish a connection between the observables at different spacetime scales $\lambda \in \mathbf{R}_+$. The action of the isometry group of $(\mathcal{M}, g)$ as well as the net structure of $\mathcal{A}$ can be lifted to $\underline{\mathcal{A}}$.
The subset of physical states in the dual space $\mathcal{A}^*$ of $\mathcal{A}$ fixes a weak topology on $\underline{\mathcal{A}}$ which allows one to proceed to arbitrarily small spacetime scales, $\lambda \to 0$. The resulting scaling limit is described by C^*-dynamical systems $(\mathcal{A}^{(0)}, \alpha^{(0)})$ in the fibers of the Minkowskian tangent bundle of $(\mathcal{M}, g)$. For the physically important case of Minkowski spacetime the general structure of $(\mathcal{A}^{(0)}, \alpha^{(0)})$ has been completely determined and classified.
Of particular interest are the outer local endomorphisms of $(\mathcal{A}^{(0)}, \alpha^{(0)})$ since they contain information on the symmetry properties of the original dynamical system at small spacetime scales. It has been shown that these morphisms fit into the setting of the Doplicher–Roberts duality theorem, and thus give rise to a compact "gauge group" $G^{(0)}$ and "field net" $(\mathcal{F}^{(0)}, \alpha^{(0)})$ which extends $(\mathcal{A}^{(0)}, \alpha^{(0)})$. These results can be used to give a precise mathematical meaning to intuitive physical notions such as "color symmetry" and "quark confinement". Part of these results is joint work with R. Verch.