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11. Mathematical Physics

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Extended Dynamical Systems

The theory of extended dynamical systems deal with the time evolution of systems where the spatial extension is important. This is a natural generalization of dynamical systems in finite dimensional phase space which were successfully used to model physical situations with only few degrees of freedom excited. A system can be thought as extended when the size of the spatial domain where it is defined is large compared to the size of the structures exhibited by the solutions. A simple example is given by the waves on a sea whose wavelength is much smaller than the sea size. In this situation it is natural to try to approximate the spatially finite system by an infinite one (this is analogous to the thermodynamic limit of statistical mechanics). The most challenging problem is to explain the origin of the structures (with their well defined scale), and to understand their time evolution. This is an area of high activity among experimental and theoretical Physicists.
We will discuss three questions which have been investigated recently from a rigorous point of view, mostly for dissipative systems inspired by hydrodynamic problems (namely non-linear partial differential equations of parabolic type).
The first problem is to prove that the evolution semi-flow is well defined for all times. The novelty of the problem is that one wants to work with solutions which are only bounded (and regular) but which do not necessarily tend to zero at infinity (for example waves). Therefore the problem of existence and regularity of solutions of the partial differential equations governing the time evolution has to be re-investigated with new methods.
The second problem deals with the occurrence of instabilities when the time evolution depends on a parameter. There is of course an analogy with standard bifurcation theory, however in the present context, when one looks at the linearized evolution around a basic solution (often homogeneous) one finds a continuous spectrum. When a relevant parameter varies, this spectrum may penetrate into the right half plane and the simple solution becomes unstable. Instabilities provide a mechanism for the appearance of solutions with well defined structures. Amplitude equations have been determined which are analogous to (truncated) normal forms. They describe the large space-time modulations of the structures near the threshold.

The third problem deals with the understanding of large time evolution. Some results have been derived for particular classes of solutions (weakly non-linear stability, interaction of structures). We will also give some results about a possible quantitative approach to the description of the so-called space-time chaos which may occur when the structures disappear or become disordered.