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11. Mathematical Physics

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Stability of matter in classical and quantized fields

Stability of matter is the statement — rooted in quantum mechanics and electrostatics — that the energy per particle has a lower bound which is independent of the state of matter and, in particular, of its amount. This fact is responsible for preventing matter from shrinking indefinitely in size, as well as for basic thermodynamic properties such as extensivity.
In recent years considerable activity was directed to the issue of stability in the case of matter interacting with an *electromagnetic field*. We shall review the results which have been established by various groups, in different settings: relativistic or non-relativistic matter, classical or quantized electromagnetic fields. Common to all of them is the fact that electrons interact with the field both through their charges *and* the magnetic moments associated to their spin. It is in fact the inclusion of spin which makes the problem of stability distinctively different from its counterpart with no electromagnetic field present. More precisely, spin allows for electron states which have no kinetic energy (zero modes) w. r. t. suitable inhomogeneous magnetic fields. This amounts to invalidating the uncertainty principle, which is however restored, in a sense, by taking into account both the kinetic energy of the electron *and* the field energy.
Stability of matter in presence of magnetic fields requires that $Z\alpha^2$ (where Z is the largest nuclear charge in the system) as well as the fine structure constant α itself, do not exceed some critical value. This is related to the fact that in a zero mode the remaining non-zero energies of the Coulomb interaction and of the field exhibit the same scaling under dilatations. If one imposes an ultraviolet cutoff to the field, as it occurs in unrenormalized quantum electrodynamics, then stability no longer implies a bound on α, $Z\alpha^2$. We conjecture that it should do so again in a properly renormalized theory.
An important technical tool, which is however of independent interest, is given by Lieb–Thirring type inequalities for the sum (or some other moment) of the eigenvalues of a one–particle Schrödinger, or related, operator. We shall present such estimates for the case of a Pauli operator with an arbitrary inhomogeneous magnetic field. The bounds involve the field strength and/or of that of the field gradient. As a corollary, we obtain a bound on the density of zero modes in a given magnetic field.