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Rogers-Ramanujan identities: A Century of progress from mathematics to physics

In this talk I will summarize the discoveries of the last five years in the theory of Rogers-Ramanujan identities; discoveries which have been made because of the interchange of ideas between mathematics and physics. This interchange of ideas is quite remarkable because when L. J. Rogers discovered his celebrated identities in 1894 there was not one shred of reason to suppose that they had anything to do with physics. Furthermore the brilliant developments by Ramanujan, Schur, Watson, Bailey, Slater and Andrews and many connections with number theory and combinatorics which were made in the first three quarters of the century contained not one sentence concerning quantum mechanics, statistical mechanics, or physics experiments. The relation between Rogers-Ramanujan identities and physics was first seen by Baxter in 1981 when he found that the original identities of Rogers appeared in order parameter computations for the hard hexagon model. A second glimpse of the relation is seen by combining the relation between conformal field theory and Virasoro algebra established by Belavin, Polyakov and Zamolodchikov in 1984 with the 1988 suggestion of Kac that every representation of the Virasoro algebra should produce a Rogers-Ramanujan identity.

Conformal field theory and, equivalently, representations of Virasoro Algebra are commonly studied by the techniques of bosonisation and this leads to compact expressions for the characters. However, there is an equivalent description of the Hilbert space in terms of fermions which leads to q-series representations of the characters and the identity of these two representations leads to Rogers-Ramanujan identities. This fermionic representation has a direct physical interpretation in terms of particles which the bosonic representations does not possess. The fermionic representation is a generalization of fermi statistics in quantum mechanics.

In the past five years we have found methods of proof which exploit these ideas to prove Rogers-Ramanujan identities for large classes of conformal field theory models including the minimal models $M(p,p')$, many coset models, and models with $N=1$ and $N=2$ supersymmetry. We have found that for cases such as the cosets $(A_1^{(1)})_1\times(A_1^{(1)})_1/(A_1^{(1)})_2$ and $(E_8)_1\times(E_8)_1/(E_8)_2$ which both describe the same CFT have different but equal q-series representations of the same characters. These different coset constructions lead

to different integrable perturbations and thus it may be conjectured that each fermionic representation of a character leads to a different integrable perturbation. I will present the current status Rogers-Ramanujan identities for the integrable $\phi_{1,3},\ \phi_{2,1},\ \phi_{1,5}$ and $\phi_{1,2}$ perturbations and will present examples where we have more identities than known integrable perturbations of conformal field theory. I will conclude with a discussion of the relation of the quasi-particle representation of the conformal field theory with the massive integrable perturbations and lattice models and with some remarks on extensions to the much larger space of nonintegrable perturbations.