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11. Mathematical Physics

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Metastability and the Ising model

This address will survey recent progress on a classical non-equilibrium statistical mechanics problem, in the context of a well-studied idealized interacting particle system, called kinetic Ising model. The problem concerns the speed and the patterns of relaxation of statistical mechanical systems in the proximity of the phase-transition region, and is related to the problem of understanding the metastable behavior of systems in such regions.
For instance, it is well known that a ferromagnetic material which is in equilibrium under a negative external magnetic field relaxes to equilibrium very slowly after the magnetic field is switched to a small positive value. A detailed mathematical analysis of such a phenomenon can only be performed on simplified models. It is widely accepted that an appropriate model for this and many other purposes is a kinetic Ising model: a Markov process which endows the traditional Ising model with a reversible stochastic dynamics. On each vertex of an infinite lattice Z^d, we have variables (called spins) which take the values -1 or $+1$. The system evolves in continuous time as a Markov process which is time-reversible and has as invariant measures the canonical Gibbs measures of statistical mechanics. When the "temperature" parameter appearing in the definition of the model is small enough, there is a phase transition which takes place when the "external field" parameter, h, changes sign (this corresponds to the change from a negative to a positive orientation of most spins). The question then arises of how the system relaxes to equilibrium when h is small (positive say), and the system is initially in the configuration with all spins -1.
In equilibrium the spins have to be mostly $+1$, with the -1 spins forming finite clusters in a background of $+1$ spins. Simulations have shown that the relaxation mechanism is driven by the behavior of the clusters (droplets) of $+1$ spins which are formed in the Sea of -1 spins. While small droplets tend to shrink and disappear, large ones tend to grow and are responsible for the relaxation. This phenomenon has long been understood on non-rigorous heuristic grounds, and can be used to predict for instance the order of magnitude, as $h \searrow 0$, of the relaxation time for the process. The prediction is that the relaxation time grows as $\exp(\lambda/h^{d-1})$, where λ is a constant which can be computed. The value of λ is, in particular, related to the equilibrium surface tension of the Ising model through the Wulff construction, which solves a variational problem. In this talk I will describe an approach which led to rigorous proofs of results of this type and other related ones.