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Space of local fields in integrable field theory and deformed Abelian integrals

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In this talk we consider the restricted sine-Gordon model (RSG) which coincides with a massive perturbation of minimal model of conformal field theory (CFT). The space of states of the model is the Fock space of particles (H_p). The space of local fields (H_f) is the same as in conformal case: it is a direct sum of tensor products of Verma modules of two Virasoro algebras corresponding to right and left chiralities (for simplicity we consider one chiral sector). The highest vectors correspond to primary fields. The Verma modules in question are reducible, the space of independent local fields ($\widehat{H}_f$) is obtained by factorizing over submodules of "null-vectors". The chiral sector of minimal models of CFT provides a quantization of Korteweg-de Vries hierarchy. The procedure of factorizing over the null-vectors corresponds to imposing the equations of motion.
Every operator in the RSG model is determined by complete set of its matrix elements in the space of states H_p. For a local field the matrix elements taken between vacuum and n-particle states are called form factors. Due to the crossing-symmetry the form factors actually define all other matrix elements, i. e. they define the local field. Putting it differently, we can say that the form factors provide a pairing between H_p and H_f. Analyzing the explicit formulae for the form factors we find the following unexpected and mathematically attractive feature of this pairing. The form factors are described in terms of deformed hyper-elliptic integrals. Different states of particles correspond to different "differentials" while different local operators correspond to different "cycles" in these integrals. Moreover, one finds the same number of possible "cycles" as the number of linearly independent vectors in the space H_f. The problem arises of explaining the origin of null-vectors in this context.
The deformed hyper-elliptic integrals satisfy identities similar to those satisfied by usual hyper-elliptic integrals. Most remarkably, there is a deformation of the Riemann bilinear identity. As a result of these identities certain local operators vanish which are shown to correspond to null-vectors in H_f. Calculating the characters we find that there is correct number of null-vectors.

Part of these results is joint work with D. Bernard and O. Babelon.

References:

F. A. Smirnov, On the deformation of abelian integrals, *Lett. Math. Phys.* **36** (1996) 267

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