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Scaling Limit of Particle Systems, Incompressible Navier-Stokes Equation and Boltzmann Equation

Although the basic microscopic physics laws of the classical or quantum mechanics are well established by the Newton or the Schroedinger equations, the macroscopic physics are described by phenomenological equations such as the Euler equation, the incompressible Navier-Stokes equation or the Boltzmann equation. Our goal is to understand mathematically the connection between these two descriptions by taking scaling limits of the microscopic equations with suitably chosen scalings of space, time, and other macroscopic parameters of the systems. Furthermore, we would like to study the corrections or fluctuations to these phenomenological equations and other problems intrinsic to the microscopic descriptions such as metastability and behavior of tagged particles. Our understanding of the Newton or the Schroedinger equation with many degrees of freedoms, however, is extremely limited and rigorous treatments of the scaling limits of many-body dynamics from the first principles are far beyond the current technique. There are three possible directions:

(A) Classical dynamics of one or a few particles in a given environment such as a classical Lorentz gas or some billiard problems.

(B) Quantum dynamics of one or a few particles in a given environment such as a quantum Lorentz gas.

(C) Stochastic dynamics of many-body systems, i. e. , the classical dynamics are modified by adding certain noise (elliptic operators).

The approach (C) retains the many-body feature while (A) and (B) treat the first principle dynamics. In this talk, we shall review some results related to (B) and (C).

We shall first describe a stochastic particle system based on random walks with collision rules on the standard cubic lattice $\mathbb{Z}^d$, $d = 3$ and state precisely a law of large numbers stating that, for initial distributions corresponding to arbitrary macroscopic L^2 initial data, the distributions of the evolving empirical momentum densities are supported entirely on global weak solutions of the incompressible Navier-Stokes equations. Furthermore the viscosity will be characterized by the Green-Kubo formula and variational principles. The key analytic inputs are a method solving the fluctuation-dissipation equation of the lattice gases and multiscale estimates

based on the logarithmic Sobolev inequality. We shall also review some applications to these methods including an estimate on the mixing rates of random rotations on S^N (Kac model of Boltzmann equation) and $SO(N)$ as $N \to \infty$.
Our second result concerns the quantum Lorentz gases which describe one electron in an environment given by random impurities. There are two relevant limits: the Boltzmann-Grad limit (or the low density limit) and the weak coupling limit. Our result states that the phase space density of the quantum evolution defined through the Wigner transform converges weakly to a linear Boltzmann equation globally in time. The collision kernel is given by the quantum differential scattering cross section in the Boltzmann-Grad limit and by its semi-classical approximation in the weak coupling limit. Notice that in the Boltzmann-Grad limit it is the true quantum differential scattering cross section that appears in the Boltzmann equation, instead of its semi-classical approximation. Thus the Boltzmann-Grad limit is not a semi-classical limit. Our method is based on a renormalized perturbation expansion with cutoff and works for a general class of statistical laws of the random impurities including Gaussian and Poisson distributions.