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Abstracts of Plenary and Invited Lectures

Section:

12. Probability and Statistics

1991 MS Classification:
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Bayesian Density Estimation

Consider a sequence of independent real valued random variables $\{X_i\}$ with unknown probability density f_o. A Bayesian makes inference about f_o by putting a prior probability measure on the space of all densities $\mathcal{P}$ equipped with a topology and a Borel structure. The posterior probability measure of the random f given the observed $X_1, \cdots, X_n$ is the Bayesian's summary. In particular, the posterior expectation of f is the Bayes estimate of f_o. Recently, various methods depending on different priors, namely, Dirichlet mixtures of normals and Gaussian processes for $\log f$, have become computationally feasible and are beginning to be widely used even though the posteriors cannot be written down explicitly. Unlike non-Bayesian methods they do not have a problem of choosing an optimal bandwidth and can adapt to the location of f_o.

A natural question of validation arises. Does the Bayes estimate converge to f_o or the posterior to δf_o in various natural senses of convergence ? Answer to such questions, for finite dimensional $\mathcal{P}$, which go back to Laplace, is usually yes. In infinite dimensional problems, like density estimation, the answer is often no. These problems are hard because explicit formulas for posterior are rare. We will explore recent positive results in the general area of Bayesian non-parametrics with main focus on the problem of density estimation. In particular, we will provide validation for the most popular method of Bayesian density estimation. Kolmogorov's metric entropy for a function space plays an important role in he development.

The talk will be technically self-contained.