ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

12. Probability and Statistics

1991 MS Classification: 62E20, 11P21, 60F05
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Lattice Point Problems and the Central Limit Theorem in Euclidean Spaces

The distribution of sums of n independent identically distributed random vectors taking values in an d-dimensional Euclidean space is approximated by a multivariate Gaussian distribution on regions defined by quadratic forms like ellipsoids and hyperboloids. Provided that the random vectors have finite fourth moments and non degenerate covariance operators error bounds of order $\mathcal{O}(n^{-1})$ are derived for regions and distributions centered at zero and dimensions $d \geq 9$. The rate is sharp improving previous bounds of order $\mathcal{O}(n^{-d/(d+1)})$ by Esseen (1945). For non centered problems the approximation is given by an Edgeworth expansion. For $d \geq 12$ upper and lower bounds of the error are of order $\mathcal{O}(n^{-1})$ and depend explicitely on the 12 largest eigenvalues only (joint work with V. Ulyanov). Comparable upper bounds are proved for the error in asymptotic expansions for the distribution of degenerated U-statistics.

Fourier inversion together with discretization and conditioning techniques reduces the problem of proving error bounds to the investigation of weighted multivariate trigonometric sums of quadratic forms on bounded sets in $\mathbb{Z}^d$. One of the basic tools is an inequality which extends Weyl estimates for such trigonometric sums and allows to control their products at different frequencies.

These techniques are applied to the estimation of the the lattice point remainder, i. e. the difference between the number of lattice points in $\mathbb{Z}^d$ and the corresponding Lebesgue volume of an open ellipsoid centered at zero and scaled by a factor R. It is of order $\mathcal{O}(R^{d-2})$ resp. $o(R^{d-2})$ for general resp. *irrational* ellipsoids and $d \geq 9$. The rate is sharp and improves previous bounds of order $\mathcal{O}(R^{d-2d/(d+1)})$ for general ellipsoids by Landau (1915). As a consequence the maximal gap between successive values of a positive definite *irrational* quadratic form tend to zero for values growing to infinity. This has been conjectured by Davenport and Lewis (1972) for all dimensions $d \geq 5$.

The results extend to hyperbolic cases like the asymptotic approximation of the number of lattice points in a cube of side length $R \to \infty$ which result in values of an irrational *indefinite* form in a fixed interval by the Lebesgue volume. For $d \geq 9$ this yields explicit bounds in terms of trigonometric

sums for the convergence in this quantitative Oppenheim problem. This convergence has been proved by Eskin, Margulis and Mozes (1997) for signatures (p, q), $p \geq 3$, $q \geq 1$ using ergodic theory on groups.
These results are joint work with V. Bentkus.