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12. Probability and Statistics

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Branching processes, random trees and superprocesses

We will survey a number of recent developments concerning the genealogy of branching processes, together with their applications to measure-valued processes and interacting particle systems, as well as some connections with semilinear partial differential equations.

A Galton-Watson branching process gives at time k the number of individuals in the k-th generation of a population where each individual gives rise, independently of the others, to a random number of children distributed according to a given offspring distribution. The genealogy of such a branching process is obviously described by a finite collection of discrete genealogical trees. One may ask about a similar description of the genealogy of continuous-state branching processes, which are the continuous analogues of Galton-Watson processes. Roughly speaking, continuous-state branching processes are obtained as scaling limits of Galton-Watson processes, starting from a "large" initial population and considering the evolution over a "long" time interval. Their distribution is characterized by a branching mechanism function ψ playing the same role as the offspring distribution of the discrete case. In the special case of Feller's diffusion ($\psi(u) = u^2$), it has been known for some time that the genealogical structure is coded by linear Brownian motion, in the sense that the contour process of the tree (describing the motion of a particle that "runs up and down along the branches of the tree") is a Brownian excursion. This fact was made explicit by Aldous, who has studied the corresponding continuous genealogical tree known as the Continuum Random Tree (CRT). In a joint work with Le Jan, we show that, for a more general continuous-state branching process, the continuous genealogical tree can be coded by a functional of a Lévy process (i. e. a real-valued process with stationary independent increments) with no negative jumps, whose Laplace exponent is precisely the branching mechanism ψ. This fact provides a unified perspective on parts of the theories of Lévy processes and of branching processes.

Measure-valued branching processes, also called superprocesses, combine a branching phenomenon (described by a continuous-state branching process) with a spatial motion, which can be for instance Brownian motion in $\mathbf{R}^d$. Roughly speaking, this means that the (infinitesimal) individuals of the continuous population move independently in space according to the law of

Brownian motion. Superprocesses have been studied extensively in the last ten years, in particular by Dawson, Dynkin and Perkins. The genealogical structure of continuous-state branching processes provides a powerful tool for investigating path properties of superprocesses as well as their connections with partial differential equations. The case $\psi(u) = u^2$ in particular leads to the Brownian snake construction of quadratic superprocesses. A typical analytic application (joint work with Dhersin) is a nonlinear Wiener test characterizing those domains in $\mathbf{R}^d$ where there exists a positive solution of $\Delta u = u^2$ with infinite boundary conditions. Other relations between superprocesses and partial differential equations, including the classification of positive solutions of $Lu = \psi(u)$ in a general domain, have been studied recently by Dynkin and Kuznetsov.

If time permits, we will briefly describe connections of the previous developments with other areas of probability theory and mathematical physics. This applies in particular to the asymptotics of certain problems of statistical mechanics. Derbez and Slade have shown that the Continuum Random Tree arises in scaling limits of lattice trees in high dimensions, and very recently Hara and Slade proved a similar result for the infinite incipient cluster in critical percolation on the lattice. Finally, a recent joint work with Bramson and Cox has established that the asymptotic behavior of the classical voter model can be analysed in terms of the genealogical structure of Feller's diffusion.