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Abstracts of Plenary and Invited Lectures

Section:

13. Combinatorics

1991 MS Classification: 5, 14, 20
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Multisegment duality, canonical bases and total positivity

The traditional interaction of representation theory, algebraic geometry and algebraic combinatorics involves:

- finite-dimensional representations of symmetric and general linear groups;
- geometry of flag varieties and Schubert varieties;
- combinatorics of Young tableaux and related algorithms.

New concepts in representation theory such as Lusztig's canonical bases [8] and Kashiwara's crystal bases [5] require new geometric and combinatorial tools. The new triad includes:

- representations of reductive groups over p-adic fields and of their Hecke algebras; quantum groups and their canonical bases;
- geometry of quiver representation varieties and totally positive varieties;
- combinatorics of rational polyhedral convex cones and polytopes, their lattice points, and their piecewise-linear transformations.

We illustrate this interplay with a particular piecewise-linear involution, the multisegment duality. It was introduced in [12, 13] in the context of representations of general linear groups over a p-adic field. It also naturally appears in the geometry of quiver representations, and in the study of canonical bases for quantum groups. On the combinatorial side, it is closely related to Schützenberger's involution on Young tableaux [11], as demonstrated in [2]. In a joint work with H. Knight [6], a closed formula for the multisegment duality was obtained using the machinery of network flows. In this talk, we give a new combinatorial interpretation of the multisegment duality as an intertwining map between two piecewise-linear actions of the Lascoux-Schützenberger plactic monoid [7]; this leads to a generalization and a new proof of the Mœglin-Waldspurger recursive description [10].

The multisegment duality is a special case of Lusztig's piecewise-linear transition maps between various combinatorial parametrizations of the canonical basis for the unipotent part of the quantum general linear group. In a joint work with A. Berenstein and S. Fomin [1], closed formulas for these maps were obtained using a parallelism discovered by Lusztig [9] between canonical bases and total positivity. Combinatorial tools used in [1] include a geometric presentation of the reduced decompositions for the symmetric group by pseudo-line arrangements, and the tropical semiring (or $(\min, +)$-algebra) appearing in combinatorial optimization and control theory. The classical theory of totally positive matrices and its recent generalization to an arbitrary reductive group due to Lusztig [9] give rise to other algebraic and combinatorial problems. If time allows, some results in this direction obtained in [1, 3, 4] will be presented.

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