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14. Mathematical Aspects of Computer Science

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Games, Complexity Classes, and Approximation Algorithms

Game theory provides a framework in which to model and analyze conflict and cooperation among independent decision makers. Many areas of computer science have benefitted from this framework, including artificial intelligence, distributed computing, security and privacy, and lower bounds. Games are particularly important in computational complexity, where they are used to characterize complexity classes, to understand the power and limitations of those classes, and to interpret the complete problems for those classes.

A standard example of the use of games to characterize complexity classes is the well-known result of Chandra, Kozen, and Stockmeyer [CKS] that Alternating Polynomial Time (APTIME) is identical to Polynomial Space (PSPACE). An alternating, polynomial-time machine and an input x of length n determine a zero-sum, perfect-information game for which the game tree has depth polynomial in n. The referee R is a deterministic, polynomial-time machine that reads an entire played game before deciding whether the winner is Player P_1 (who claims that x is in the APTIME set L) or Player P_2 (who claims that x is not in L). P_1 (resp. P_2) has a winning strategy if and only if x is in L (resp. x is not in L). Because P_1, P_2, and R are deterministic, R always decides the winner correctly; in game-theoretic terms, the value of the game is 1 if x is in L, and it is 0 if x is not in L.

Subsequent results of Papadimitriou [P], Babai and Moran [BM], Lund, Fortnow, Karloff and Nisan [LFKN], and Shamir [S] show that the connection between PSPACE and zero-sum, perfect-information, polynomial-depth games is quite robust. Both [P] and [BM] modify the basic computational model of APTIME so that Player P_2 uses a trivial strategy: Whenever it is his turn to move, i. e., to write down a $p(n)$-bit string for some polynomial p, P_2 simply writes down the results of $p(n)$ fair coin-flips. Now if P_1 plays optimally on input string x, there is some probability (computed over the coin tosses of P_2) that R declares P_1 to be the winner; that probability is the value of the game determined by x, P_1, and R. In Papadimitriou's "games against nature, " x is in L if and only if the value of the game is at least 1/2. In Babai and Moran's "Arthur-Merlin games, " x is in L if the value of the game is at least 2/3, and x is not in L if the value is at most 1/3. Under both definitions, the class of sets accepted by such a game system is still PSPACE. (This was proven in [P] for games against nature and in [LFKN] and [S] for Arthur-Merlin games.) Thus we get the same computational complexity class from zero-sum, perfect-information, polynomial-depth games whether P_2 is a strategic player or a random player and whether we demand that the referee compute the value of the game exactly (as in games against nature) or approximately (as in Arthur-Merlin games).

In this talk, I will present two recent characterizations of PSPACE that push its identification with zero-sum, perfect-information, polynomial-depth games even further. The Probabilistically Checkable Debate System model (PCDS) is obtained by modifying the APTIME model in two ways: The polynomial-time referee R is allowed to flip coins, and it is not necessarily allowed to read the entire transcript of a played game before deciding whether the winner is P_1 or P_2. If x is in L, then R must decide in P_1's favor with probability 1, and if x is not in L, then R must decide in P_2's favor with probability at least 2/3. In the Randomized PCDS model (RPCDS), Player P_1 and referee R are exactly as in PCDS, but Player P_2 must play by flipping a sequence of fair coins. Condon, Feigenbaum, Lund, and Shor [CFLS1, CFLS2] show that a set is in PSPACE if and only if it is recognized by a PCDS and by an RPCDS each of which allows its referee to flip $O(\log n)$ coins and to read only a constant number of bits of the game transcript.

If time allows, I will present two related set of results about games and complexity. The first uses these characterizations to show that certain natural maximization and minimization functions, drawn from domains such as propositional logic, graph searching, graph reliability, and stochastic optimization, are as hard to approximate closely as they are to compute exactly. The second is a general framework defined by Feigenbaum, Koller, and Shor [FKS] and further developed by Feige and Kilian [FK] in order to explore relationships between complexity classes and imperfect-information games. Some tight characterizations of exponential-time complexity classes are obtained, but no class of imperfect-information games can be as robustly identified with any complexity class as perfect-information, polynomial-depth games can be with PSPACE.

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