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Section:

15. Numerical Analysis and Scientific Computing

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Lloyd N. Trefethen, Oxford University, UK, and Tobin A. Driscoll, University of Colorado, USA
Schwarz–Christoffel Mapping in the Computer Era

In the past twenty years, because of new algorithms as well as new computers, Schwarz–Christoffel conformal mapping of polygons has matured to a technology that can be used at the touch of a button. Many authors have contributed to this progress, including Däppen, Davis, Dias, Elcrat, Floryan, Henrici, Hoekstra, Howell, Hu, Reppe, Zemach, and ourselves. The principal SC software tools are the Fortran package SCPACK [5] and its Matlab successor, the Schwarz–Christoffel Toolbox [2]. It is now a routine matter to compute an SC map involving a dozen vertices to eight digits of accuracy in a few seconds on a workstation.
With the power to compute has come the ability to explore. The obvious kind of problem—"here is a polygon; map it onto a disk or a half-plane"—is rarely what one encounters in practice. Instead it is variations on the idea of SC mapping that arise. The published paper will describe four examples:
(1) oblique derivative problems on polygons [6]
(2) ideal two-dimensional jets, wakes, and cavities [1]
(3) the CRDT (cross-ratio Delaunay triangulation) algorithm [3]
(4) Green's functions for multiply connected domains with symmetry [4].
References will be given to variants of SC mapping including rectangles, infinite strips, periodic domains, fractals, circular polygons, curved boundaries, gearlike domains, exterior polygons, doubly connected polygons, and polygonal Riemann surfaces. Applications mentioned will include grid generation, Faber polynomials, polynomial approximation, digital filtering, matrix iterations, the KdV equation, electrical resistances and capacitances, magnetostatics, the classical Hall effect, queuing theory, vortex methods in fluid mechanics, and inverse problems.

References

[1] F. Dias, A. R. Elcrat and L. N. Trefethen, Ideal jet flow in two dimensions, *J. Fluid Mech. 185* (1987), 275–288.
[2] T. A. Driscoll, Algorithm 765: A MATLAB toolbox for Schwarz–Christoffel mapping, *ACM Trans. Math. Softw. 22* (1996), 168–186.
[3] T. A. Driscoll and S. A. Vavasis, Numerical conformal mapping using cross-ratios and Delaunay triangulation, *SIAM J. Sci. Comput.* , to appear.
[4] M. Embree and L. N. Trefethen, manuscript in preparation.

[5] L. N. Trefethen, Numerical computation of the Schwarz–Christoffel transformation, *SIAM J. Sci. Stat. Comput. 1* (1980), 82–102.
[6] L. N. Trefethen and R. J. Williams, Conformal mapping solution of Laplace's equation on a polygon with oblique derivative boundary conditions, *J. Comp. Appl. Math. 14* (1986), 227–249.

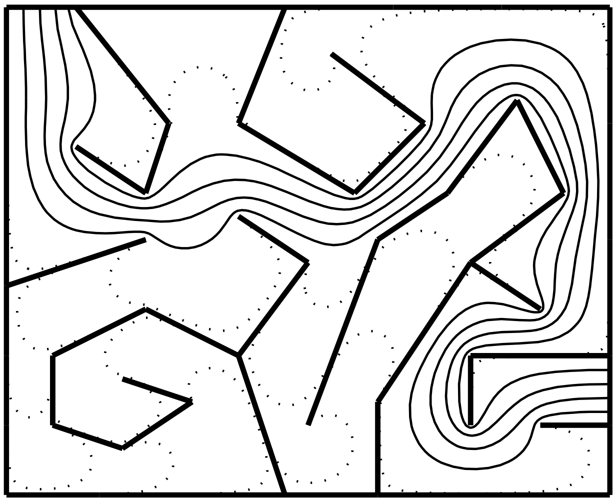

Figure 1. Conformal map of "Emma's maze" onto a rectangle, computed by the CRDT algorithm [3]. The solid curves are the conformal images of straight lines in the rectangle; the dotted curves are the same, but exponentially close to the rectangle boundary. Because of exponential distortions, the numerical computation of maps like this is far beyond the capabilities of ordinary SC algorithms.

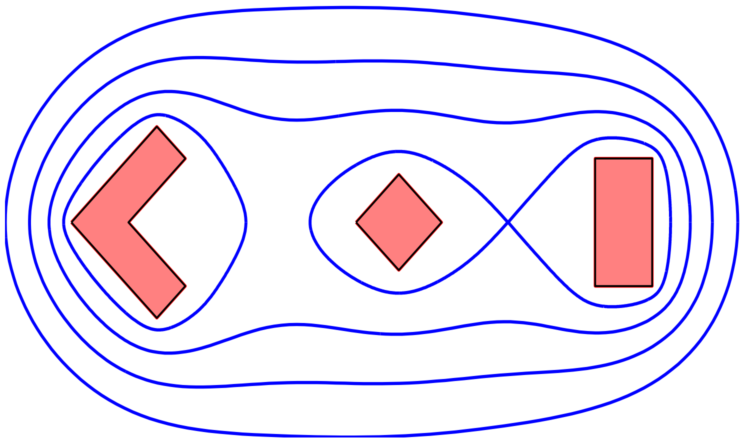

Figure 2. Level curves of the Green's function for a multiply connected domain [4]. Because of the symmetry with respect to an axis, the Green's function can be computed via a conformal map onto the exterior of a disk with two protruding radial slits. The level curves are images of concentric circles.