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16. Applications

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Canonical Models in Mathematical Neuroscience

Experiments have shown that there are phase relationships between inputs and outputs in neurons [1]. To study these, we model neural systems in the frequency domain. Neural systems are sometimes at rest and sometimes oscillating, and there is a hierarchy of canonical models describing how they can change from being at rest to oscillating [2]. For example, the Saddle-Node on Limit Cycle (SNLC), the Andronov-Hopf (AH) and the Saddle-Loop (SL) bifurcations describe such phenomena. The first two have co-dimension 1 and the second co-dimension 2. In SNLC and SL an oscillation of (almost) constant amplitude but increasing frequency appears, and in AH an oscillation of increasing amplitude and (almost) constant frequency appears. Both phenomena are observed in neuroscience experiments [3]. We focus here on SNLC and SL cases using the VCON model (voltage controlled oscillator neuron model [4]), which is based on phase-locked loop electronic circuits. We show that populations of such oscillators self-organize by frequencies [5]; that a same-frequency sub-population of oscillators can interact in the sense that a change in phase deviation in one will be felt by the others in the sub-population [2]; and that oscillators operating at different frequencies do not interact in this way. Therefore, sub-networks are defined by the firing frequency of their constituents. Elements can change their sub-population membership by changing their frequency, much like tuning to a new station on an FM radio. Thus, a rich array of network configurations is possible without rewiring. Specifically, the dynamical system

$$\tau\ddot{\theta}_j + \dot{\theta}_j = \omega_j + \cos\theta_j + (1-\cos\theta_j)\sum_{k=1}^{N} C_{k,j}(1+\cos\theta_k)$$

for $j,k = 1,\cdots,N$, describes a network of N oscillators having center frequencies ω_j, voltage outputs proportional to $1+\cos\theta_j$, and connection strengths between oscillators $C_{k,j}$, indicating the strength of connection from site k to site j. Each of these phase variables has an asymptotic limit of the form $\theta_j \to \rho_j t + \phi_j$ where ρ_j is the asymptotic frequency (rotation number) and ϕ_j is the asymptotic phase deviation. A sub-network comprises the set of oscillators having the same value for ρ_j, and for this set information is carried by the phase deviation ϕ_j of each. This model, an analysis and computer simulations of it, and some related issues will be described.

References

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