ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

16. Applications

1991 MS Classification: 65M12, 76B15
Hou, Thomas Yizhao, California Institute of Technology, USA
Numerical Study of Free Interface Problems Using Boundary Integral Methods

Many physically interesting problems involve propagation of free interfaces. Water waves, boundaries between immiscible fluids, vortex sheets, Hele-Shaw cells, thin-film growth, crystal growth and solidification are some of the better known examples. Numerical simulations for interfacial flows play an increasingly important role in understanding the complex interfacial dynamics, pattern formations, and interfacial instabilities. Many numerical methods have been developed to study these interfacial problems, including phase field models, volume of fluids, level set methods, front tracking methods, and boundary integral/element methods. Here we will focus on boundary integral methods.
Numerical study of fluid interfaces is a difficult task due to the presence of high frequency numerical instabilities. Small perturbations even at the round-off error level may lead to rapid growth. This makes it very difficult to distinguish the numerical instability from the physical one. In our study, we first establish the well-posedness of the linearized motion far from equilibrium. This involves careful analysis of singular integral operators defined on free interfaces. The continuous well-posedness analysis provides a critical guideline for our numerical analysis of the discrete system. We found that there is a corresponding compatibility condition between the discrete singular operators and the discrete derivative operator. Violation of this compatibility condition will lead to numerical instability. We completely eliminate the numerical instability by introducing an effective filtering. The amount of filtering is determined by enforcing a discrete compatibility condition. The resulting scheme is shown to be stable and convergent. The corresponding 3-D problem is considerably more difficult since the singular operators have non-removable branch point singularities and there is no spectrally accurate discretization. A new stabilizing technique is introduced to overcome this difficulty. This technique is very general and effective. It also applies to non-periodic problems with rigid boundaries.
Surface tension also introduces a severe time step stability constraint. We overcome this difficulty by reformulating the problem in tangent angle/arclength (curvature in 3-D) variables and using a small scale decomposition technique. Using this reformulation, the leading order singular operators become linear and have constant coefficient. Thus semi-implicit

discretizations can be easily applied and can be inverted efficiently. This greatly improves the time step stability constraint for a number of interfacial problems. Applications to Hele-Shaw flows, 3-D vortex filaments, and the Kirchhoff rod model for protein folding all give very impressive results. The improved method enables us to perform a careful numerical study of the stabilizing effect of surface tension for fluid interfaces. Water waves with small surface tension are shown to form singular capillary waves dynamically. The mechanism for generating such capillary waves is revealed and the zero surface tension limit is studied. In another study, surface tension is shown to regularize the early curvature singularity induced by the Rayleigh-Taylor instability in a unstably stratified two-fluid interface. However, a pinching singularity is observed in the late stage of the roll-up. The interface forms a trapped bubble and self-intersects in finite time.
Motivated by our analysis, we propose a new model equation for interface problems in both two and three dimensions. This model equation is nonlocal and nonlinear. But the nonlocal operator is given by the Hilbert transform in 2-D, and by the Rietz transform in 3-D. Consequently they can be evaluated fast with $NlogN$ operations by Fast Fourier Transform. Our numerical experiments have demonstrated convincingly that our model equations can captures effectively the small scale features even for interfaces with curvature or corner singularities.