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16. Applications

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Iooss, Gérard, Institut Universitaire de France, Valbonne, France

Travelling water-waves, as a paradigm for bifurcations in reversible infinite dimensional “dynamical” systems

First we recall few properties of reversible systems, in finite or infinite dimensional spaces, of the form

$$\frac{dU}{dx} = F(U), \qquad SF(U) = -F(SU), \quad S^2 = \mathbb{I}$$

near a symmetric equilibrium taken at 0. Considering bifurcation problems, i. e vector fields depending on a set of parameters, which are non hyperbolic near 0 for certain parameter values, we give necessary conditions under which *center manifold reduction* applies for the study of solutions staying close to the equilibrium for all $x \in \mathbb{R}$ (see the basic paper [6] for application of center manifold reduction to elliptic problems in a strip). We give the basic example of "*reversible Hopf*" bifurcation and state the results on known small bounded solutions: periodic, quasi-periodic, homoclinic to 0, homoclinic to periodic solutions. The main tool for such a study is Poincaré-Birkhoff normal form theory [1] (revisited following the global characterization given in [2], taking care of reversibility) and a proof of persistence of reversible (symmetric) solutions under higher order terms, not considered in the normal form [5] (see [3], [7] for other typical proofs on a similar reversible bifurcation).
We then present a large class of water-wave problems, where we look for 2D travelling waves in a potential flow, with one or several superposed layers, in the presence or not of surface tension at the free surface and at interfaces. Coordinate x corresponds to the direction of travelling waves. In case of finite depth layers, such a problem of finding small bounded solutions for all $x \in \mathbb{R}$, is shown to be reducible to a finite dimensional symmetric center manifold, on which the system reduces to a reversible ODE. For "*reversible Hopf*" bifurcation, this leads to various kind of travelling waves, for instance to solitary waves with damped oscillations at infinity or to "black" solitary waves, corresponding to solutions homoclinic to periodic waves with a minimum of amplitude at $x = 0$.
If the bottom layer has infinite depth, which appears to be the most physically realistic case, concerning the validity of results in the parameter set, the mathematical problem is far more difficult, because it cannot be reduced to a finite dimensional one, due to the occurence of a continuous spectrum

of the linearized operator $DF(0)$ crossing the imaginary axis (at 0). This continuous spectrum may also contain an embedded eigenvalue at 0. We give some ways, via an infinite dimensional normal form reduction, on how to attack this difficulty, specially for periodic and homoclinic solutions with a *polynomial decay* at infinity (see [4] for solitary waves in the case of one layer of infinite depth, in presence of surface tension).

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