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17. Control Theory and Optimization

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Open Dynamical Systems and their Control

Dynamical systems usually interact with their environment. Such systems are called *open systems*. It is often essential to incorporate this feature in the mathematical model, for example, in application areas such as signal processing and control engineering. The aim of this talk is to describe the mathematical structures that emerge in modelling open dynamical systems, to discuss two central concepts, controllability and observability, and to apply these ideas to the design of robust controllers. For ease of exposition, we will concentrate on a rather specific class of systems, linear time-invariant differential systems.
We now describe our formulation of the control problem in a bit more detail. I n a typical control problem we have a given plant that relates three (vector-valued) time-signals: disturbances d, to-be-controlled variables z, and control variables c. Control is viewed as interconnecting a controller to the plant, with the controller acting only through the control variables. Before the controller is attached to the plant, the variables $w = (d, z)$ define a set $\mathcal{P}$ of realizable time-signals; $\mathcal{P}$ is, for example, a linear shift-invariant subspace of $\mathfrak{L}_2^{\ell oc}(\mathbb{R}, \mathbb{R}^q)$. When the controller is attached to the plant, it restricts $\mathcal{P}$ to $\mathcal{K} \subseteq \mathcal{P}$: $\mathcal{K}$ is called the *controlled behavior*. The fact that the controller can act on the control variables only, translates into the requirement that $\mathcal{K}$ must contain a given subspac e $\mathcal{N}$ of $\mathcal{P}$. The control problem is thus to find, for a given
$\mathcal{N}$ and $\mathcal{P}$, a controlled behavior $\mathcal{K}$ with $\mathcal{N} \subseteq \mathcal{K} \subseteq \mathcal{P}$ such that $\mathcal{K}$ meets certain specifications. For example, in robust control, $\mathcal{K}$ must leave the exogenous disturbances d free, it must be stable (appropriately defined), and it must be attenuate the disturbances ($|| \ z \ || \leq || \ d \ ||$ in suitable norms).
The solution to this robust control problem involves the theory of linear dissipative systems with quadratic functionals. We will present this solution and sketch algorithms that allow to compute the controlled behavior and the controller in terms of a representation of the plant in terms of a differential equation.