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18. Teaching and Popularization of Mathematics

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Aspects of the nature and the state of research in mathematical education

During the past three decades, a rapidly growing body of research on a multitude of aspects of mathematics education has contributed to establish what today is an academic discipline sometimes referred to as 'the didactics of mathematics', sometimes simply as mathematics education research. What are the issues and research questions in this discipline, what is its methodology, what sorts of findings does it offer, and what sorts of evidence does it provide, in particular with respect to matters of importance to university mathematics?

In this lecture, attempts will be made to characterise the didactics of mathematics as a discipline, and a number of major findings will be presented and discussed. More specifically, it will be explained why such findings only relatively seldom take the shape of empirical or experimental results in the traditional sense (and never, of course, of mathematical theorems). Nevertheless, research findings in mathematics education resulting from thorough theoretical or empirical analyses give rise to solid insights of considerable significance for our understanding of processes and outcomes of mathematical teaching and learning, and hence for the ways in which mathematics may, or may not, be successfully taught and learnt. As far as major findings are concerned, the ones to be presented in this lecture have been selected after consultation of a fair number of the leading researchers in mathematics education in the world.

One major finding may be formulated as follows, *the key role of domain specificity*: For a student engaged in learning mathematics, the specific nature, content and extent of a mathematical concept that he or she is acquiring or building up are to a large part determined by the set of specific domains in which that concept has been concretely exemplified and embedded for that particular student. At first sight, this may seem to be little more than a reformulation of a well known observation belonging to the experience of any observant and reflective teacher of mathematics at whichever level. (If this is true, which it may well sometimes be, it is remarkable, however, how often the finding remains unobserved in actual teaching practice.). But at closer inspection, the range and depth of the instances of this finding have far-reaching bearings on the teaching and learning of mathematics. Thus, not only are most 'usual' students unable to grasp an abstract concept in and of itself unless it has been elucidated by multiple examples (which is well known), but, more importantly, the scope of the notion a student forms is often barred by the very examples given to support that notion. This implies that it is a non-trivial matter of teaching and learning to establish abstract mathematical concepts so as to be both sufficiently general and sufficiently concrete in their scopes. Research further suggests that for this to happen several different representations (e. g. verbal, symbolic, graphical, diagrammatical, etc,) of concepts and phenomena are essential, as are the links and transitions between these representations.

The above-mentioned finding becomes particularly intricate with those mathematical concepts which are formed as objects generated by and encapsulating certain processes. Simple examples of this are the concept of function (arising from mechanisms that produce the values of the function) and the concept of derivative (arising from the process of differentiating). This process-object duality is referred to in the research literature by different terms, including 'procept' (an amalgamation of process-concept), 'tool-object' or 'reification'. This constitutes another major finding in the didactics of mathematics, *the obstacles produced by the process-object duality*. This duality gives rise to fundamental learning (and teaching) hurdles that are difficult to overcome for students (and teachers). Many students have substantial difficulty in leaving the process level and enter the object level. For example, many students conceive an equation as signifying a request to perform certain operations, without dis- tinguishing these operations from the concept of equation as such. To them, an equation simply does not constitute a mathematical entity. Research even shows that quite a few of those who can actually solve a given equation correctly are unable to subsequently tell correctly whether or not a given element is a solution to the equation or not. Other students are able to establish notions of both the processes underlying a certain concept and that concept as an object, but have difficulty in establishing links between the two. The theory of so-called 'conceptual fields' offers general perspectives on these matters.

The findings sketched above are connected to yet other major findings, such as *the mathematical knowledge of the individual learner is not necessarily hiearchical* (it is possible to possess correct knowledge of a certain segment of mathematics without possessing correct knowledge that may be considered a logical or a conceptual prerequisite for the former knowledge), *students' conceptions of basic aspects of mathematics are often rational, even if they are incorrect or insufficient*, a phenomenon which is sometimes explained by the fact that some students' initial, but implicit, conceptual models are so strong and robust that they exert a decisive influence on later learning experiences.

These and other findings (concerning issues such as students' conceptions of proof; problem solving; assessment; the influence of technology; the significance of the cultural and social environment, of the context, and of the students' activities, on the learning outcomes) will be considered in the lecture.