ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

19. History of Mathematics

1991 MS Classification:

Karine Chemla, CNRS and Universite Paris VII, research group REHSEIS, France

History of Mathematics in China: A factor in World History and a Source for New Questions

In the last decades much research has been devoted to the *Jiuzhang suanshu* or *The nine chapters on mathematical procedures* (hereafter abbreviated to *The nine chapters*), a book which played a crucial part in the mathematical traditions which expressed themselves in Chinese, quite comparable to the one played by Euclid's *Elements of geometry* in the West. Compiled around the beginning of the common era, after the unification of the Chinese empire, the book was soon to become a "Classic" from which most subsequent Chinese mathematicians drew inspiration. As for all the other writings granted the status of Classics in China, commentaries were composed on *The nine chapters* , some of which were selected by the tradition to be handed down together with the book. This is how commentaries ascribed to Liu Hui (third century) and Li Chunfeng (seventh century) survived until today.
The talk will present some recent results obtained on the book itself and its commentaries. It will then discuss various aspects of the transmission of mathematical knowledge between China and the West. Finally, the talk will c onsider some new questions for the study of mathematical activity raised by research on *The nine chapters* . In conclusion, it is argued that only an international approach to history of mathematics can provide an adequate framework to capture the historical processes which have constituted mathematical lores around the world.

I. Recent Developments in the History of Mathematics in Early Imperial China

The nine chapters are constituted of problems and general algorithms with which to solve them. Inspired by Donald Knuth, who suggested reading Old-Babylonian clay-tablets from the point of view of algorithmic theory, Wu Wenjun initiated a new approach to ancient Chinese mathematical sources along similar lines. Algorithms found in *The nine chapters* such as root extractions, rules of false double position, or algorithms to solve systems of simultaneous linear equations, evidence the use of sophisticated resources: assignment of variables, conditionals, iterations. They demonstrate the development, wit hin an algorithmic framework, of an algebra that I shall characterize. In contrast with *The nine chapters* themselves, the commentaries explicitly set out to prove the correctness of the algorithms provided by the Classic. This practice of proof will be described. In particular, it will be argued that here we find, again within an algorithmic context, the development of a form of algebraic proof. The commentator Liu Hui stresses the connection between this form of proof and the various kinds of numbers introduced by the Classic (fractions and quadratic irrationals). How both aspects relate to a philosophical outlook on mathematics developed within the commentaries will also be outlined.

II. A Factor in World History

The earliest extant Chinese and Indian mathematical sources demonstrate enough basic common features to state that, from early on, the communities practising mathematics in both areas must have been part of a common intellectual world. Later on, Arab scholars became interested in this scientific world through India. This is documented historically. However, several elements common to Chinese and Arabic sources from the 9th century onwards, and so far not found in the known Indian sources, seem to indicate that there were direct contacts between the Chinese- and the Arabic-speaking intellectual communities. Using *The nine chapters*, t he talk will hint at several cases of circulation of knowledge, for the different perspectives they provide on the processes thus constituting mathematical knowledge, and on the situations they created.

III. New questions

Not only do ancient Chinese sources contain original results, some of which are now part of common knowledge in world mathematics. They also evidence a specific way of practising mathematics, which can be recovered through historical analysis. For instance, the counting board on which Chinese mathematicians computed was handled according to strict and particular rules. So me recurring features of mathematical notations in China can be traced back to that practice of computing, and the nature of some basic concepts seems to originate from observing mathematical reality as given shape through computations on the board. It will be argued that all the elements entering into the practice of mathematics d́1problems, visual aids such as figures, literary techniquesd́1 demonstrate specific features in the way mathematicians used them in ancient China. However, this is not peculiar to China: more generally, the products of mathematical activity that eventually become universal are worked out by resorting to specific forms of practice, partly inheri ted, partly reworked according to the problems addressed or the conditions available. Clearly, describing such regimes of mathematical activity and their relations to the mathematical results produced, is history with an agenda, and I shall conclude by discussing its relevance.

Bibliography

K. Chemla and Guo Shuchun, *The nine chapters on mathematical procedures.* A Critical edition, translation and presentation of the classic and the commentaries on it ascribed to Liu Hui (third century) and Li Chun-

feng (7th century), Diderot multimedia (to appear).