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Secretariat
Centre for Mathematics Education
University of Southampton
Southampton, SO9 5NH
England

The International Commission on Mathematical Instruction

BULLETIN NO. 20

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Editors: Keith Hirst and Geoffrey Howson
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University of Southampton
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INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

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Please note the following ammendments to the list of National Representatives.

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ICM, BERKELEY: AUGUST 1986

The following is the program for ICMI sessions at Berkeley.

Sunday, August 3

17:00 School Mathematics in the 1990's.
Speaker: D. Wheeler

Chairman: B. Nebres

Monday, August 4

17:00 Mathematics as Part of a Liberal Education at the University Undergraduate Level.
Speaker: S. Goldberg

Report on Worldwide Mathematics Education. How can ICMI help?
Speaker: H. Whitney

Chairman: H.O. Pollak

Tuesday, August 5

17:00 The Southeast Asia Mathematical Society (SEAMS), its Work and Problems.
Speakers: Lee Peng Yee and Lim Chong Keang

Chairman: L. Steen

Wednesday, August 6

17:00 The International Mathematical Olympiads (IMOs) from Their Origin to the Present Time.
Speaker: Cl. Deschamps

Panel Members: H. Alder and M. Hussein

Chairman: L.D. Faddeev

Friday, August 8

17:00 The U.S. Mathematical Sciences Education Board.
Speaker: Shirley Hill

Chairman: A. Gleason

Saturday, August 9

17:00 Mathematics Education in the People's Republic of China.
Speaker: Ding Er-sheng

Chairman: Z. Semadeni

Sunday, August 10

17:00 Mathematics as a Service Subject.
Speaker: J. van Lint

Future ICMI Activities.
Speaker: J-P. Kahane

Chairman: M.F. Newman

In addition the International Study Group on the Relations between the History and Pedagogy of Mathematics will mount a meeting on the theme 'advances in mathematics and their incorporations into collegiate and university curricula: a history perspective and implications',on Friday 8 August from 7 p.m. to 11 p.m.

ICME 6

The list of Chief Organisers of Theme and Action Groups is now complete. To the names given in Bulletin 19 should be added

A.5 Tertiary/post-secondary/academic institutions (age 18+).

John Mack, Department of Pure Mathematics, The University of Sydney, N.S.W. 2006, Australia.

T.1 The profession of teaching.

Peggy House, Department of Curriculum and Instruction, College of Education, Peik Hall, University of Minnesota, 159 Pillsbury Drive SE, Minneapolis, Minnesota, 55455, USA.

STUDIES IN MATHEMATICS EDUCATION

Volume 3 The mathematical education of primary-school teachers (1984)

Volume 4 The education of secondary school teachers of mathematics (1985)

Robert Morris (ed.) Paris: UNESCO

Most of the various 'international' journals in mathematics education inevitably reflect some bias towards the prevailing issues in their country of publication, a situation recognised and accepted within the profession. By contrast, a journal produced by UNESCO has to pay particular attention to balancing the interests of mathematics educators in different regions, working in different languages and at different school levels, and within the frameworks of differing educational traditions. The advantages of achieving such a balance are in terms of the cross-cultural fertilisation of ideas and practice from which the reader may unexpectedly learn; the danger is that contributions may be disparate and the total content lacking in coherence.

Volume 1 of 'Studies', published in 1980, was such a pot pourri, consisting of surveys of current practice in seven countries covering a wide range of economic and educational development. The need for a consistent theme within an issue was recognised in Volume 2 (1981) where the contributions examined national goals for mathematics education in the light of the trend towards mathematics-for-all.

The coherence has been further developed in Volumes 3 and 4, both of which concern the education of mathematics teachers. Volume 3, devoted to the primary school level, recognises that in most countries primary teachers are generalists, and so mathematics is but one component among many in their training and in their subsequent responsibilities. Accordingly it is of paramount importance that the priorities for this component are clearly set, and observed by teacher educators; there simply is not time to cover everything that one would like to do with the trainee primary teacher. Volume 3 of 'Studies' assists in the identification of these priorities and examines how they are being put into practice in a wide variety of educational systems which span the obvious divides: developed/developing, rich/poor, Africa/Asia/the Caribbean/Europe/Latin America/North America/Pacific. The nature of primary mathematics is first considered as the background to later chapters on teacher education, both pre-service and in-service. Of particular challenge to many educators is the chapter which strongly advocates the widespread intelligent use of calculators in primary classrooms as soon as a country can afford it.

In Volume 4 the attention is on the secondary school mathematics teacher, who is assumed to be a subject specialist. In some ways this makes the task of the teacher educator easier, although the issues within schools become more sharply focussed: the great diversity of achievement of students, the statistical disparity of performance between boys and girls, the ever-pressing question of the relevance of school mathematics to employment and to the mathematical demands of citizenship, the impact of calculators and computers. These and other issues are helpfully discussed in contributions by 17 authors from 10 countries. Of course, discussion is one thing, putting conclusions into practice another. Recognising this, the series editor Robert Morris commissioned two case studies which form the final two chapters. One is from Zimbabwe, the other - of particular interest because the material is relatively unknown - from China.

It is encouraging to learn that, despite recent regrettable defections from UNESCO, 'Studies' will continue at least for the forseeable future. Mathematics educators in many countries will welcome this decision and will continue to benefit from the journal's catholicity and the accessibility of the writing.

Bryan Wilson

FORTHCOMING INTERNATIONAL MEETINGS

1-5 September, 1986.	'Classroom Observation Methods for Curriculum Development', Hugh Burkhardt, Shell Centre for Mathematical Education, University of Nottingham, Nottingham, NG7 2RD, England.
6-10 April, 1987.	'Mathematics as a Service Subject' International Centre for the Mechanical Sciences, Udine, Italy.
13-18 April, 1987.	'Renovation de l'Enseignement des Mathématiques et Informatique dans les Pays en Developpement' Yamoussoukro, Côte d'Ivoire, (Société Mathématique de Côte d'Ivoire, 08 B.P. 2030, Abidjan 08, Côte d'Ivoire)
17-19 May, 1987.	'Informatics and the Teaching of Mathematics' Sofia, Bulgaria, (Peter Bollerslev, Røjlevangen 40, DK2630 Taastrup, Denmark)
1-3 June, 1987.	Fourth S.E. Asian Conference on Mathematical Education, Singapore. (Dr. Ong Sit Tui, c/o Mathematics and Computer Studies Department, Institute of Education, 469 Bukit Timah Road, Singapore 1025.)
12-16 July, 1987	Seventh Inter-American Conference on Mathematics Education Santo Dominigo, D.R.
27 July-3 August 1988	ICME 6 Technical University, Budapest, Hungary.

LINKING UNIVERSITY MATHEMATICS EDUCATORS

In many countries University departments or Schools of Education each contain only a small number of mathematics educators. In some cases, there may be only one such person in a University, with the attendant problems of isolation, lack of professional sharing and difficulties with inducting new staff. Links with colleagues doing similar work elsewhere offers the potential for stimulus, the spread of ideas and the development of relevant skills.

An awareness of this situation and need in the U.K., led to the establishment of an informal study group of mathematics education tutors some twenty years ago. It has grown in scope and size since then, now existing under the title Association of University Mathematics Education Tutors (AUMET). Interest in its work has been expressed by a number of colleagues in other countries and a few have become members. Some members of AUMET are interested to know if similar associations exist in other parts of the world. If so, some communication or informal link between associations might be mutually beneficial. It is hoped that this brief note about AUMET might set some productive activity in motion.

In the U.K. about 30 Universities have Schools, Faculties or Departments of Education. Almost all are engaged in providing one year initial training courses for graduates who intend to teach in secondary schools (11-18 years). Some provide 4 year concurrent degree/initial training courses and some train graduates for primary school teaching. A major commitment of most University mathematics educators is to initial education in theory and practice. However over the last twenty years there has been a growth in other kinds of activity particularly that relating to inservice training of various kinds. Substantial academic courses leading to masters degrees (M.A. M.Ed.) are taught and research-work leading to M.Phil and Ph.D. is supervised. More recently University departments, along with other training institutions, have provided courses for heads of mathematics departments from local secondary schools, as well as programmes of short courses for local primary and secondary teachers. As might be expected in Universities, another area of work relates to research and development. This ranges from individuals working in self selected specialist fields to small teams funded for limited periods on issues considered to be of local or national significance.

In a working environment which features multiple demands and some freedom for individual initiative, it is inevitable that a rich variety of practice and experience has developed. AUMET's activities have sought to make use of this for the benefit of the 100 or so tutors who are working in such an environment in the U.K. Its principal activities are

(i) an annual 2/3 day residential conference

(ii) an annual research day based in a different University each year

(iii) a newsletter

(iv) representation on various national bodies e.g. The Royal Society's Mathematics Education Committee, Joint Mathematical Council

(v) providing a communication network for members

(vi) occasional participation in development activity in relation to teacher training.

If this note should come to the attention of those engaged in running similar associations, we shall be very glad to hear from you and to know of your activities. I shall be happy to provide more information about AUMET to any such organisation and indeed, to any individual who cares to contact me.

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35 Berkeley Square,
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SIXTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

23-27 November 1985, Guadalajara, Mexico.

Organized by the Inter-American Committee for Mathematics Education with the support of Unesco and hosted by the University of Guadalajara, this congress attended by nearly 200 mathematics educators from 24 countries discussed four main themes: historical and cultural aspects in the teaching of mathematics; changes in mathematics teaching due to the influence of calculators and microcomputers; mathematical modelling and problem-solving; and identification of causes of failure in school mathematics.

The proceedings, largely in Spanish, will be published by the Unesco Regional Office for Science in Montevideo. The next (seventh) congress will be convened in the Dominican Republic in 1987, organized by the Universidad Catolica Madre y Maestra. Further information may be obtained from:

> Dr. Eduardo Luna, Universidad Catolica Madre y Maestra
> Autopista Duarte, Km 1 1/2, Santiago de los Caballeros
> Dominican Republic.

ICMI SYMPOSIUM

on

'The Influence of computers and informatics on mathematics and its teaching'

We reproduce in this issue an account written by Rolf Biehler, Rudolf Strässer and Bernard Winkelmann of IDM, Bielefeld.

We are grateful to the Editors of Zentralblatt für Didaktik der Mathematik for allowing us to reprint this article and hope that readers will find that its interest will help compensate for any eye-strain!

Report on the ICMI-Symposium on "The influence of computers and informatics on mathematics and its teaching"

Strasbourg, 25–29 March, 1985

Rolf BIEHLER, Rudolf STRÄSSER, Bernard WINKELMANN;
IDM Bielefeld

1. Preparation and procedure of the symposium

1.1 Motives and aims of the symposium

"The International Commission on Mathematical Instruction (ICMI) is planning a number of studies on topics of international interest. Each study will be built around an international seminar and will be directed towards the preparation of a published volume intended to promote discussion and action at national, regional or institutional level" ([1], p. 159). This was the introduction to the discussion document on "The influence of computers and informatics on mathematics and its teaching" which served as the key-preparatory document of the symposium to report on.

The document contained a general introduction to the ICMI-studies and the study on computers and informatics, including remarks on the procedure of the study. The document itself termes the advent of the computer as the "second industrial revolution", which implies "new needs, new sciences, new technologies, new qualifications, the elimination of repetitive or laborious work, and of course, new social challenges to be met". The document is "organized around three important questions: 1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science? 2. How can new curricula meet the needs and possibilities? 3. How can the use of computers help the teaching of mathematics? As far as questions 2 and 3 are concerned, we are limiting our study to the curriculum and teaching at university and pre-university level (from the age of 16 years)" ([1], p. 161).

The document was published early in 1984 in "L'enseignement mathématique" and was additionally circulated to the national commitees of ICMI. In some cases – e.g. the FR Germany – it was more or less by chance that interested persons came across the document if they were not regular readers of "L'enseignement mathématique".

The discussion document already included a call for reactions to the document – and this call for papers led to 41 papers which were compiled as "Document de Travail" (DocTrav [2]), and distributed to all participants a couple of weeks before the conference. Some additional papers were distributed during the symposium thus forming some 50 reactions to the discussion document to be taken into account as related to the study during the symposium.

The symposium was planned and held mainly to discuss these papers and additional presentations at the symposium (cf. part 2) in order to have the discussion document elaborated on a broader basis (cf. the description of the "working groups" in part 2.8–2.10). Revised versions of the papers are to be found in a collection of "Supporting papers" (SupPap [3]).

1.2 Preparation, participants and place of the symposium

As participation of the conference was only by invitation, the planning committee (presumably a subset of the authors of the discussion document: R. F. CHURCHHOUSE, B. CORNU, A. P. ERSHOV, A. G. HOWSON, J. P. KAHANE, J. H. VAN LINT, F. PLUVINAGE, A. RALSTON, M. YAMAGUTI) asked persons from around the world to participate in the symposium – preferably those who had submitted papers. Some 70 participants from Australia (1 participant), Belgium (5 participants), Canada (3), FR Germany (9), Finland (1), France (14), Italy (2), Ivory Coast (1), Japan (3), Malaysia (1), Netherlands (3), United Kingdom (11), USA (14) and from UNESCO (1 representative) made the symposium an international forum of discussion. The participants can be characterised as preferably belonging to the mathematics scientific community whereas a minority could be described as mathematics educators by profession.

The symposium took place at the Centre Saint-Thomas in Strasbourg, March 25–29, 1985. It was a stimulating experience with oral and visual presentations, lively discussions and an output worthwhile reading (cf. part 3 of the report).

1.3 Procedure of the symposium

The symposium started working on Monday, March 25 in the afternoon, and had its closing session on Friday, March 29, 1985 late in the afternoon.

Three types of activities could be discerned during the symposium: Plenary sessions, lasting for half a day, were chaired by a person who was designated to concentrate on a topic identified. Monday afternoon (chair: J. P. KAHANE) concentrated on the influence of informatics and computers on mathematics as a discipline; Tuesday morning (chair: A. RALSTON) was on discrete and continuous mathematics; Wednesday morning (chair: H. POLLAK) on problems of implementation and visualization; Thursday morning (chair: J. VAN LINT) on symbolic algebra and logic and Friday morning (chair: L. A. STEEN) put together interventions on geometry, statistics and various educational matters. The presentations in the plenary sessions varied in length and relatedness to the topics indicated above. Most presentations were followed by discussions on the presentation or related subjects.

Three "working groups" tried to elaborate the discussion document by generating comments on the three chapters of the document (ICMI 84 [1]). These working or "writing groups", as they were called at the symposium, met as parallel groups on Tuesday, Wednesday and Thursday afternoon. Each working group produced papers commenting on the topics "influence on mathematics as a discipline" (chair: R. CHURCHHOUSE), "influence on mathematics curricula" (chair: A. RALSTON) and "influence on mathematics teaching and learning" (chair: B. CORNU, for details cf. part 2.8 to 2.10 of the report).

The evenings were devoted to presentations and lively discussions of computer programs, software packages and video tapes or transparencies of computer-produced visualizations related to the topics of the symposium. They illustrated the variety and complexity of the symposium's subject and made it an enterprise different from many other international meetings.

2. Topics of the symposium

In this part of the report, we shall give some more detailed information on what really was presented and discussed during the symposium. This information will be organized along the structure of the plenary sessions and working groups of the symposium, but will also include references to papers to be found in [2] and [3]. The evening presentations will also be referred to if appropriate.

2.1 Influence on mathematics as a discipline

The ICMI background paper already contained several ideas concerning the influence of computers and informatics on mathematics. Two essential aspects of the influence were emphasized: the *tool aspect* (computers as tools for doing mathematics) and the *application aspect* (computers and informatics as a domain, where mathematics is applied to, and as a source of mathematical problems).

Some examples were given to substantiate the statement that "computers and informatics have *stimulated* new research, *restored* to the mathematicians' consideration questions recently neglected but previously studied over a long period of time, and *made possible* the study of new questions" ([1], p. 163).

This rather abstract characterization was supplemented by identifying several specific features of the computer's influence: a new emphasis and extension of the experimental side of mathematics, new possibilities of visualizations, an effect on the status of proof, an emphasis on algorithms, and changes related to the use of symbolic manipulation systems.

The plenary session on Monday afternoon consisted of a mixture of short 10 minutes long presentations of papers from the DocTrav by their authors (R. BIEHLER, L. A. STEEN, M. YAMAGUTI), and additional talks on the 4-colour-theorem by B. MORIN and on Julia sets and fractals by B. MANDELBROT und B. WEST.

The session started with discussing the status of the computer aided proof of the 4-colour-theorem given by K. APPEL and W. HAKEN. The ICMI background paper described the change as follows: Until computer-aided proof emerged, a mathematician working alone was supposed, in principle, to be able to follow and to verify every step of a proof thanks to the usual method of presentation. This is no longer possible with computer-aided proofs like the 4-colour-proof. They have produced the need, therefore, for a new form of professional practice ([1], p. 164).

M. F. ATIYAH (U. K.) agrees that the proof was a great triumph, but "from an aesthetic point of view, it is extremely disappointing, and no new insights are derived from the proof. . . . Can we be said to 'understand' the proof of the 4-colour-theorem? I doubt it" ([2], pp. 5). He then raises the question whether more and more problems will be solved by "brute force" in the future and expresses the opinion that "Mathematics is really an Art – it is the art of avoiding brute-force calculation by developing concepts and techniques which enable one to travel more lightly. Give a mathematician an infinitely powerful machine for doing calculations and you deprive him of his inner driving force" ([2], p. 6).

L. A. STEEN (USA) comments that despite "philosophical alarms that computer-based proofs change mathematics from an *a priori* to a contingent, fallible subject . . ., careful analysis reveals that nothing much has really changed. The human practice of mathematics has always been fallible; now it has a partner in fallibility" ([3], p. 24).

Given this background, the plenary session was opened by a critical review of the 4-colour-theorem by B. MORIN (France). He emphasized the point of view that the 4-colour-theorem is no traditional proof at all, because it relies on an "experiment" on a computer. It should not be accepted that mathematical truth relies on the correct functioning of machines. Additionally, proof is a way of understanding, of illumination, and the 4-colour-proof does not fulfill this criterium.

This somewhat extreme opinion stimulated interesting discussions. Some argued that certainty should not be the main problem, computer-aided proofs could be checked with different machines and programming languages. In fact, such a check might give more certainty than a check by mathematicians. The critique that the proof is not illuminating was considered to be more important. But would mathematicians also argue for teaching only those proofs in the secondary classroom that are illuminating to the students? was a question asked by H. BURKHARDT. Also, it was said that "illumination" and "understanding" might be too subjective to serve as a basis for the acceptability of proofs.

Another key concept in the discussions and presentations was *experimental or exploratory mathematics*. The ICMI background paper pointed to this aspect emphasizing that there has always been an experimental side to mathematics. The computer has greatly increased the possibilities for observation and experimentation. M. F. ATIYAH ([2], p. 4) expresses his point of view as follows: "In Mathematics, as in the Natural Sciences, there are several stages involved in a discovery, and formal proof is only the last . . . In all the earlier stages computers can play a role." As described above, he is more critical towards any essential use of computers in proofs.

Whereas these opinions seem to emphasize the continuity in the change, the following contributors seem to emphasize the new qualitative nature of the change.

M. YAMAGUTI (Japan), reflecting on his own research, where the computer is an essential tool, puts it as follows: The computer changed the structure of the evolution of mathematical concepts. He mentions computer experimentation and especially visualization: "The merit of this kind of visualization is enormous, because by the visualization of some possible facts, mathematicians in many different fields have a common avenue of research. This causes a recovery of the unity of mathematics" ([3], p. 17). He says this with regard to experiences concerning the so-called soliton solution of non-linear wave equation, and to the study of chaotic dynamical systems. If YAMAGUTI's observation is more generally true, one could perhaps say that the computer makes it possible to "objectify" the experimental side of mathematics. B. MANDELBROT (USA) and B. WEST (USA) illustrated the essential importance of graphical displays and numerical experimentation in the study of fractals and Julia sets ([3], p. 297). In an evening presentation MANDELBROT illustrated his short talk by showing dozens of beautiful transparencies.

In a sense, fractals and Julia sets formed a paradigm for the conference about what may be achievable with the help of computers. An additional reason for this appreciation may be found in the fact that this topic is deeply rooted in the history of mathematics and, in the course of research, very interesting and unexpected relations to other mathematical areas were established.

Now, B. MANDELBROT himself used his experiences for an attack against the so-called "doctrine of pure mathematics", i.e. the view that mathematics can and should make progress by employing "pure reason" and free invention alone. He emphasized that using the visualizing and experimental capacities of computers helped to solve problems (by looking at mathematical objects or mathematical models of natural phenomena and experimenting with them) which pure reason alone was not able to do for one hundred years (e.g. in the area of random motion).

As another domain where the influence of computers shows up very clearly, statistics was mentioned. In his presentation, R. BIEHLER (FR Germany) refered to an important revival of graphical methods as a research tool in statistics and to changes in the methodology of statistics, which are often summarized under the key word "Exploratory Data Analysis". In these areas, experimental mathematics has a somewhat different meaning. A formal proof is not the final goal. E.g. statistical techniques are explored and compared by means of computer simulation in situations where analytic methods and proofs are not available. Also, the experimentation with several transformations and displays of data helps to generate conjectures about the underlying system, conjectures that are more similar to hypotheses in the sciences than to conjectures in pure mathematics (cf. [3], pp. 209).

2.2 Discrete and continuous mathematics and questions of calculus

The approach of the conference steering group to organize the complexity of the conference's themes neatly according to the headings of the preparing paper (roughly speaking) as mathematics, curricula, and teaching methods led to a certain partitioning of the contributions to the teaching and learning of calculus over the last two points which was only partly accepted by the conference participants.

Calculus appeared in the conference

- explicitly in the plenary session on Tuesday morning under the heading „discrete and continuous mathematics"
- in some scattered contributions during other sessions,
- on most of the presentations of computer programs in the evenings, and
- in a special subgroup which was scheduled as part of working group 3 ("Learning and Teaching") but established itself as a rather autonomous subgroup of working group 2 ("Curriculum") in order to be able to start early (cf. 2.9).

The chairman for the *plenary session* on Tuesday morning on *"discrete and continuous mathematics"* was A. RALSTON (USA) who is very well known in North America for his fight for a new undergraduate curriculum in mathematics at colleges and universities for mathematics majors, natural scientists and computer scientists. He votes for the inclusion of a course on discrete mathematics with the same importance as the traditional calculus courses. (Part of the corresponding debate is documented in the book RALSTON/YOUNG (eds.): The future of college mathematics, New York: Springer, 1983, which has gained attention also in Europe.) He obviously interpreted the theme of the plenary session in the sense of this North American debate. This debate seems to be influenced by "social" issues such as maintaining the size of mathematical faculties under the pressure of computer science on the one hand, and course construction and related implementation issues according to specific needs of specific groups of students on the other.

RALSTON structured the plenary session by asking the following questions:

- Should discrete mathematics replace parts of calculus?
- Can fundamental concepts of analysis be approached by discrete mathematics?
- What discrete mathematics should be introduced, taking into account the heterogenity of students needs?
- What content could be omitted from the fundamental calculus courses?

K. BOGART (USA) (cf. [3], pp. 251–254) reported results of a survey in which members of mathematics faculties were asked for content-topics to be included or excluded in a course on discrete mathematics for students beginning deeper mathematical studies at college level, especially mathematics and computer science majors at freshman/sophomore level. There seems to be some consensus on contents, mainly in accordance with some proposals of the American Mathematical Society (AMS). A typical course might have the units: sets and logic, functions and relations, mathematical induction, basic combinatorics, advanced combinatorial analysis, trees, graphs, probability, matrix algebra, symbolic logic. In the discussion, such a course was critizised as to be not coherent and on a too low level, and therefore not suitable as a replacement for calculus for the intended audience.

E. R. MULLER (Canada) (cf. [3], pp. 247–250) described experiences with a specific course in discrete mathematics which was centered around the algorithmic way of thinking and the mathematics needed by computer scientists at this very moment. Principles and tools for generating and analysing algorithms are taught, e.g. proof techniques as tools for the verification of the correctness of programs.

S. B. SEIDMAN (USA) (cf. [3], pp. 235–245) instead described a first-year curriculum that integrated continuous and discrete aspects. The rationales were to have both aspects at hand when needed rather early in the curriculum of computer scientists, taking into account that students are often not capable of combining discrete and continuous skills (e.g. to apply calculus techniques to estimate growth rates of discrete functions) if these are taught in separate courses. The course – described as mostly continuous with a strong discrete flavor – was build around the theme of functional behavior and representation combining topics from traditional (continuous) and discrete calculus fitting to this theme. Continuous functions and discrete sequences are treated in analogy, e.g. integration and summation, continuous and discrete logarithm. The developers believe the course not only to fit for computer science students, but for mathematics and engineering students as well.

In contrast B. R. HODGSON (Canada) (cf. [3], pp. 255–258) considered the new possibilities of mathematical software (numerics, graphics, symbolic calculations) and their impact on the calculus concepts and techniques. Some techniques and especially the drill in executing algorithms will loose importance. The time gained should be used for stressing the historical development, abilities for qualitative analyses and more interactive classroom teaching. Now, useless drill in calculations can be replaced by deeper understanding, thereby transmitting the idea of calculus as an intellectual achievement and not as a bag of outdated techniques and tricks. This somewhat optimistic view was partly questioned in the discussion; the only topic to be reduced in emphasis agreed upon seemed to be integration by parts, and benefits in deeper understanding from using computers were seen mainly in the realm of differential equations which hitherto has not been part of the beginning semesters. In the discussion H. BURKHARDT (U.K.) compared this to the 1960 new math reform which was also partly based on the wrong opinion that teaching few principles suffices for transmitting understanding; experience showed the clear need for extensive practice as a basis for understanding.

In the first proposal for the organization of the plenary session there were only these contributions from the USA and Canada; the European contributors (CORNU, WINKELMANN) were ad hoc invited for short presentations in order to give some international perspectives to what otherwise would have been to be considered a regional discussion.

B. WINKELMANN (F. R. Germany) (cf. [2], Supplement pp. 1–7) pointed to calculus in the European countries. Its general educational value as part of the general pre-university education is more important than questions of specific preparation for specific disciplines. The view of mathematics as an academic discipline doesn't seem sufficient for a proper determination of the calculus courses; the place of mathematics in the general culture and its changes because of the general availability of the computer should be one starting point: most mathematical applications in industry and science nowadays are mediated by the computer. Hence the range of mathematization has been blown up considerably, on the other hand, mathematics has the tendency to be hidden in its technical aspects in these applications. What should the user of mathematics know in order to use intelligently the mathematical possibilities computers and mathematical software offer to him today?

In his short presentation, B. CORNU (France) described the French approach to computer use in mathematics teaching. The classical didactical triangle "teacher – student – knowledge" is to be supplemented by the computer as a fourth element. This raises deep questions for educational research which should be tackled before giving definite answers to curricular questions.

Most of the *software presentations* in the evenings were related in one sense or another to questions of calculus. Since it was not possible to critically watch all of these presentations, we just describe here some of these which we think are important and remarkable.

D. TALL (U.K.) presented some of his ingeniously invented programs on the BBC micro (Acorn B) for the support of building calculus concepts. They generally use discrete numerical approximations which can be controlled by the user through interactive graphical display. The user is not supposed to have knowledge of programming, but nevertheless is given great opportunity to test his intuitive concepts, to easily make experiments ("what if . . ."), and to view graphics of well-chosen examples (see [3] pp. 291–295 or the March 1985 issue of the journal "Mathematics Teaching" for some pictures). The programs are written to facilitate the learning process and to avoid the limit concept at the beginning. They fit into 16 K RAM and so can be used in most English schools, but unfortunately they are not commercially available yet, and a recoding for other types of computers doesn't seem easy.

B. WEST (USA) demonstrated programs on the Macintosh computer for the visualization and qualitative study of (ordinary) differential equations (of the first degree), cf. [2], Supplement, pp. 29–36. It was impressive to see how the computational power and the graphical user interface (pull down menues, mouse to point to specific points in the graphical display and to cause specific actions such as drawing (numerically computed) solutions through the points pointed at, or magnifying parts of the display) combine to give the user possibilities of easily and rapidly exploring many qualitative aspects of a differential equation such as critical points, stability. Thus the advances in micro computer technology can actually be exploited by mathematics education!

The programs presented by M. MASCARELLO (Italy) and one of her students had a different goal. While the professionally designed programs of TALL and WEST could not have been developed by the interested mathematics teacher during some afternoons or nights, these were typically programmed by engineering students of the Politecnico de Torino as homework on dynamical systems.

2.3 Problems of implementation

In an introduction to a 90 minutes debate on problems of implementation of computer use in the teaching of mathematics, A. G. HOWSON (UK) set the scene by asking for experiences, or even a strategy for this major curricular change. In fact, the session tackled two issues: problems with the development of software for educational use and experiences with the introduction of computer use into learning processes.

As for the development of software, M. THORNE (UK) distinguishes two categories: "amateur" software "programmed by the lecturer who uses it (which) is not robust in its user interaction . . . (and) very useful to its author but highly non-exportable". As opposed to this there is "professional" software, which "is foolproof in every sense . . . and thoroughly tested before release" (THORNE, in [3], p. 52). Concentrating on professional software he argues for a "cost-effective software development" including "teaching and learning packages which many sites are able to use" ([3], p. 51). But he also identifies traps in this line of development, such as the tendency to "play safe by attempting to implement existing objectives, . . . to avoid significant innovatory developments . . . (leading away) from research to applications" ([3], p. 50). As some sort of an answer to these questions, he advocates for the introduction of computer-based tools which "assist both teaching and research" like CAYLEY for the study of finite groups ([3], p. 54).

An additional lesson to be learnt from the development of software packages was mentioned by K. D. LANE (USA) in his conference presentation: the user should be included already during the development of (educational) software. Or as M. THORNE (UK) put it: "Software houses . . . producing material for . . . school use have learnt the importance of having the teachers specify the software which is to be developed . . ., trialling and feedback . . . (which is then incorporated into the design) are absolutely essential" (THORNE, in [3], p. 53). This is turning away from the Research-Development-Dissemination-model (R-D-D-model) of curriculum development as applied for instance in the USA for the "New Math"-movement and described and criticized in A. G. HOWSON / Ch. KEITEL / J. KILPATRICK (Curriculum development in mathematics. Cambridge 1981, p. 126ff).

Giving up the R-D-D-model and bringing in the teachers in the curriculum development do not solve the implementation problems. Two additional problems were identified during the symposium. In [3] (p. 57ff) D. R. SARGENT (UK) identified problems related to software as a "major problem area" to the introduction of more computing aids into the classroom. Software "rarely meets the exact needs of the teacher . . ." and if available "it may not be available in a form which matches the hardware" at hand at a given college (SARGENT, [3], p. 58 and Suppl. to [2], p. 39). As some sort of remedy D. TALL (UK) put forward the idea of an international documentation center for educational software to be used in mathematics, while M. THORNE (UK) mentioned the importance of maintenance costs which are "by far the greatest cost incurred in developing and using a system". This is especially true for large-scale, "professional" software packages (cf. THORNE in [3], p. 53).

In his presentation, M. F. NEWMAN (Australia) described some experiences and problems with the implementation of the software package MATRIX in the teaching of linear algebra course at university level ([3] p. 195 ff). Students were offered to use MATRIX to do exercises in the tutorials. Three quarters of the students tried, and about half of those continued to use it throughout the course. A questionnaire investigated the reasons for not using the packages and brought to light the following reasons of more general importance:

a) Lack of time. The discussion suggested a cost-benefit-perspective. Learning to use an additional tool always costs time. A critical issue must be the time spent on learning to use the software; it can come from that otherwise spent in doing calculations.

b) Necessity to understand the essentials of the course. It seems to be a long-standing belief of mathematics teachers that the essential ideas in a course can be explained by simplified, artificial examples. Now it is possible to use more complex and realistic examples. Might this be more attractive or perhaps effective for a majority of the students?

c) Dislike of computers. It was not clear whether it were mainly women that disliked computers in the study. But, in any case, a negative attitude towards computers might exacerbate problems in the learning of mathematics.

d) Use in examinations. It is well known that here lies an obstacle for any innovation. The technical difficulties for arranging examinations with essential computer use are known, let alone the development of new contents for examinations. And, should one really certify the use of software?

H. BURKHARDT (U. K.) and D. R. SARGENT (U. K.) in two papers open up a broader perspective: H. BURKHARDT advocates for an "empirical approach – find out what actually happens to your draft ideas in practice, in circumstances sufficiently representative of what you are aiming for, and then revise the materials repeatedly until they work in the way intended. We have found that structured classroom observation is a key ingredient in this approach" (BURKHARDT in [3], pp. 119). This again brings out the "overwhelming importance of the people, teacher and pupils and of the dynamics of their interaction". The contribution also identifies helpful effects of classroom computer use by introducing "open elements . . . essential for teaching problem solving" ([3], p. 124) and taking over "for a time a substantial part of the teacher's normal load of explaining, managing and task setting. . . . It is equally important to recognise that there will be disappointments – or at least frustrations" ([3], p. 124) because intelligent computer use in classrooms is more demanding on the teacher than a traditional lesson ([3], p. 123). "The use of a micro requires a different strategy where the teacher becomes part of the audience and where teacher and students work together actively. Teachers may feel insecure in these transitions from formal and informal styles." In investigative activities the teacher "may not have experienced all possible parameters and so the student will need to be disabused of the notion that teacher knows it all" (SARGENT in [3], p. 40 f). Here we are back again at the teacher as the key person of the implementation of curricular change resulting from computers and informatics.

2.4 Visualization in mathematics education

A short plenary session on Wednesday morning was devoted to visualization in mathematics education. J. NIMAN (USA) reported experiences from the elementary school with Logo environments and with teaching data analysis using graphical displays. The graphical capabilities of computers may/should lead either to a shift of importance of particular topics or to the introduction of new topics like two dimensional geometry (by using Logo).

R. ALLEN (cf. [3], p. 131-137) reported on a geometry CAI project in Rennes, France. The project's objective was to support the finding and construction of proofs in geometry. During a first stage only a "proof editor" using PROLOG syntax was used besides paper-and-pencil construction of diagrams. A major step forward was to design and implement a "graphic editor" by which a direct graphical input and manipulation of figures is possible. Several experiments were made to clarify the role of diagrams for finding and understanding proofs. The discussion raised the critical question that it should be more carefully analysed what the particular advantages of using computer graphics are as compared to other media like films about geometry or drawings by hand.

Beside the new use of visualizations in mathematical and statistical research (already described in 2.1), which may also have implications for mathematics education, the following suggestions and reflections were presented.

In D. TALL'S (U. K.) (see also 2.2) approach to teaching calculus, visualizations play an important role in facilitating the learning and use of fundamental concepts. Visualizations are used to avoid formalities at a too early stage and should expand the students' mental image of the concepts, thus giving them additional *meaning*. This approach could also be viewed as a reasonable preparation for the use of typical commercial software packages which provide only results and call for greater insight into the processes that produce such results as a prerequisite for their adequate use.

Similarly, J. H. HUBBARD'S and B. WEST'S approach (cf. Supplement to [2], pp. 29 and 2.2) to using computer graphics on the Macintosh computer for teaching differential equations was motivated by the observation of learning difficulties in mathematics courses. Several advantages of computer graphics were underlined, among others: conveying visual and psychological reality to the *general validity* of the existence and uniqueness theorems; combining qualitative theory and numerical analysis; tool for digesting lists of numbers; opportunity for observing and experimenting with particular solutions as a motivating and challenging stimulus for developing a more systematic theory.

D. J. SAUNDERS (U. K.) presented and described the background ([3], pp. 157) of computer animations in mathematics teaching at the Open University (U. K.). They are important tools for improving "teaching at a distance". Simulations in main-frame computers are filmed and distributed on video and TV. This reduces the amount of interactivity as compared to the above two examples. On the other hand, visualizations become accessible that are not possible on existing micros. A main idea is to use the specific advantages of films, i.e. *changing and moving diagrams*. Examples from mappings in non-Euclidean geometry and from probability and statistics were presented at the conference.

On Friday morning, S. K. KIVELA (Finland) (cf. [3], p 277–284) presented ideas concerning the influence of computer graphics and CAD (computer assisted design) systems on the teaching of geometry, especially at Helsinki University of Technology. Preparing for the use and understanding of CAD systems calls for a new geometry curriculum in which aspects of three-dimensional geometry, like projection mappings, geometric transformations, principles for constructing models and twodimensional pictures are emphasized. The use of computer graphics as a means of visualization is underlined, but available CAD systems are judged to be not well enough suited for such general purposes.

In summary, some very convincing ideas and projects were presented. The claims concerning the educational advantages were supported by pilot trials and partly some systematic evaluations have begun or are in the planning stage. D. R. SARGENT ([3], pp. 57) reports systematic experiences and problems with using computer graphics at the Open University (U.K.):

- a new teaching style (less expository) is necessary
- the teacher must be convinced of the value of visualizations
- students should have the opportunity to use the package to reinforce the ideas it puts forward. This has implications for the userfriendliness of the package
- experience shows that students remember little from a single viewing of a TV program. That might be not so different with computer graphic packages. The follow-up activities, after "the machine has been switched off", have to be planned very carefully.

Additionally, critical voices argued that experiences with other media in mathematics education suggest that computer graphics (media in general) must have some essential value; otherwise it will not be used, but considered as a methodological luxury. Also, it should not be underestimated that, in general, computer visualizations cannot be used and understood immediately, rather they call for a more profound learning of the interpretation and manipulation of diagrams and graphs.

2.5 Computer algebra and logic

Under the chair of the Dutch mathematician J. VAN LINT, the presentations and discussions on the subject of symbolic algebra and logic provided fresh insights into these fast-developing fields which were somewhat unfamiliar to most of the conference participants.

J. DAVENPORT (U. K.) gave an introductory overview over the mathematics underlying important symbolic systems. Starting with well-known problems such as multiplication of (big) integers and polynomials, factorization of polynomials and indefinite integration, he showed how the underlying algorithms use rather advanced theorems and techniques of abstract algebra. DAVENPORT reported that he himself teaches such methods only to graduate students specializing in computer algebra. The methods cannot be understood by most intended users of such systems, while the results of such computations (e.g. integrals) are totally understandable. This raises the question of how such "black boxes" may be handled in mathematics education; whether they should be made "half-open" at least for math students by writing supplements to these programs, or whether they could be taken as granted.

D. R. STOUTEMYER (USA), the developer of the muMATH-systems, proposed some discovery activities in the mathematics classroom by the use of a computer algebra system; e.g. finding patterns in iterated formal manipulations (e.g. with summations or integrations which are just not feasible without computers) as an experimental starting point and thus gathering experiences and insights as a basis for formal proof (cf. [3], pp. 185–190). This was seen as an example of how to use a software tool for pedagogical goals which was not specifically designed for education.

K. D. LANE (USA) reported on a calculus course in which the MACSYMA-system was systematically used as a "calculating slave" in order to free the student's concentration for conceptual understanding, to make him autonomous instead of imitative, e.g. in the domain of curve sketching. The computer system takes over the part which is irrelevant to the main content of the course.

In the discussion it was underlined that traditional teaching experiences suggest that – at least for many students – some drill even in imitative activities seems necessary in order to foster understanding. On the other hand, there was hitherto really no other way to develop understanding *and* get results; as it is possible to get results by other means than doing calculations and formula manipulation ourselves, we may now perhaps even find new ways for understanding. So the problem of determinating the proper amount of drill and practice for understanding was explicitely left open as a research question for mathematics education.

At the end of this part of the session, H. POLLAK (USA) reported on the beginning of computer algebra systems and an illustrative example of their early use (in the sixties) at Bell Labs. This example is sketched in the report of working group 1 (see 2.8).

Questions of logic were only partly discussed in the plenary sessions; a specialist debate took place in a separate evening session on "Logic, computers and education", on which we cannot report. In the plenary session there were two presentations on questions of logic in mathematical education:

N. G. DE BRUIJN (Netherlands) (cf. [3], pp. 169–178) reported on Automath, a notational system which should allow the writing of mathematical books whose correctness could be verified by a computer; the necessary effect on explicitness and completeness was regarded of educational value. The discussion made clear that this presupposes a special view of mathematics and mathematical knowledge.

A similar view of mathematics was the starting point of another presentation on "what should a student in mathematics know about computer science" in order to bridge the gap between these two disciplines. It was argued in favour of the topics computability, algorithms, and syntax and semantics, which should be approached from both disciplines with their somewhat different languages and points of view.

2.6 Statistics, probability, data analysis

The first half of the plenary session on Friday morning was mainly devoted to new possibilities and shifts of emphasis for teaching probability and statistics.

A. ENGEL (FR Germany) (cf. [3], p. 215–222) presented ideas for a course in statistics that emphasizes the algorithmic standpoint. With regard to statistics, the emphasis lies on non-parametric tests, bootstrap methods and simulation, thereby taking into account new developments in statistics that were reinforced by the computer technology. Only real data are analysed, statistical tables are not necessary, P-values instead of prespecified significance levels are used. Because of the emphasis on statistical algorithms, several important computer science topics can be learned via the suggested statistics course, e.g. how to deal with rounding errors in numerical algorithms, using recursion, searching and sorting algorithms, decreasing the execution time of programs/algorithms, elements of complexity theory.

J. P. CONZE (France) (cf. [3], p. 223–225) argued for a shift of emphasis in teaching probability (mainly at University level) which is related to the effect computers had on probability theory. Simulation could be used as a way of "seeing" random variables and events to enhance intuition, instead of teaching random variables as part of measure theory. The notion of random sequence should become more important as well as its relations to ergodic systems which in turn are related to chaotic dynamical systems. Random number generators on computers are a new topic of interest from a practical and from a theoretical point of view. The relation between the theory of random sequences and the theory of recursive functions was mentioned in particular. A new emphasis on these relations of probability to other parts of mathematics may lead to deemphasizing measure theory as the traditional mathematical environment of probability.

R. BIEHLER (FR Germany) (cf. [3], p. 209–214) presented examples and methodological principles of Exploratory Data Analysis (EDA) in comparison to traditional statistics, thereby also illustrating in which sense EDA is a "child" of the computer age. Reasons for and against integrating EDA into secondary stochastics curriculum were reviewed. If a more flexible and practice-related statistics and data analysis curriculum is desirable, using computers may facilitate such an approach considerably. But there are obstacles for such an innovation, e.g. the state of pre- and inservice training, as well as teacher knowledge and attitude toward statistics. Present trends in teacher education, at least concerning computers in the FR Germany, do not support such innovations, because topics from informatics seem to be more important than approaching computers from the perspective of teaching mathematics.

H. O. POLLAK (USA) reported on experiences with teaching some simple techniques and diagrams of EDA in secondary schools in the United States. Plotting, summarizing, transforming, smoothing and comparing data, as well as drawing tentative conclusions, were central activities in which the students become engaged. It is important that the students feel that the data are their "own data" which they want to analyze. The less rigorous and less abstract subject matter of EDA, where micros help to do the plotting and calculating in reasonable amounts of time, seems to be good material for such an approach and perhaps also a good preparation for a more traditional statistics course.

The presentation stimulated a vigorous discussion. Is it desirable and feasible for mathematics teachers that statistics is taught at the secondary level at all? Statistics and in particular EDA were assumed to be so different in nature from usual mathematics. On the other hand, several participants confirmed the need for better preparing students to adequately use statistical software. This should include knowledge about the "ethics" of classical statistics and of EDA. The diversity of influences of computers on probability and statistics which was obvious from the presentations made it clear that this alone could not serve as a basis for curriculum decisions.

Perhaps the controversy can also be interpreted on the background of some papers arguing that one important influence of computers on mathematics (education) is/should be a stronger emphasis on formalisation, formal logic and precise use of syntax whereas other participants saw the educational promises of the computer in somewhat less rigorous, more intuitive and experimental mathematics.

2.7 Educational matters

The last plenary session (90 minutes) was under the heading of "experiments in informatics and mathematics" and brought together presentations from very different points of view. E. DUBINSKI (USA) reported on experiences with the introduction of computers and teachware into mathematics teaching at the undergraduate level. Starting from Piaget's theory of cognitive development, he reported on experiments with concept acquisition supported by computer experiences of these concepts offered to the students. In [3] DUBINSKI reports on three projects (cf. [3] p. 63ff): namely the first on creation and composition of functions using VENIX (a version of UNIX available on mini computers), the second project on "gathering genetic data on the development of the concept of induction" in mathematics majors which confronts them with experiences with "while-loops" in computer programs, while the third project tries to teach mathematical concepts by advising the students to think of the concept "in terms of its actual construction by the computer" using the language SETL (cf. [3], p. 64f). Computer use was found valuable and helpful, but experience also warns against a too optimistic look into future educational use of computers and teachware: "As we move into more abstract concepts in mathematics, it may become increasingly difficult to find appropriate computer experiences. This is an area that will require some creative activity and also, as we grow in our understanding of how the cognitive development takes place, we may find that experiences which do not relate to computers may be helpful in the same way" (DUBINSKI in [3], p. 69). A short additional hint to DUBINSKI'S presentation seems appropriate: DUBINSKI not only reported on computer use in education, but also described a research strategy in mathematics education: "All of the concepts in undergraduate mathematics should be analysed and their genetic decomposition determined. The curriculum should be rearranged . . . in terms of the interrelationship amongst the structures of which the concepts are composed" ([3], p. 69).

K.-D. GRAF (FR Germany) commented on the relation of the disciplines informatics and mathematics at university and secondary school level, advocating "that math education already fulfills many of the objectives which informatics has written on its flag for the schools", e.g. teaching applications, even non-numerical complex problems using the modelling approach and fostering working in a group instead of individual learning. An integrated teaching of mathematics a[nd] informatics was seen as the "right combination . . . (t[o] arm the students with effective strategies for proble[m] solving" (cf. [3], pp. 41).

M. MASCARELLO (Italy) reported on the use of micr[o] computers in a first year calculus course at the Facul[ty] of Engineering Sciences at the Politecnico of Torin[o]. The computer is used mainly as a visualization to he[lp] the learning process, thus making "more attractive a[nd] easier to teach and to learn" some topics "which we[re] traditionally considered as 'hard' and 'too abstra[ct]' . . . Moreover, we gave the basis for a numerical anal[y]sis for some problems which are not solved in a sati[s]factory way by qualitative theory, at least from t[he] point of view of a student of engineering" (cf. [p. 266).

R. STRAESSER (FR Germany) made "Remarks on t[he] interplay of mathematics, computers and society" ([3], p. 13–15) stating and illustrating that it is diffic[ult] to get scientific knowledge on society's needs f[or] mathematics education. He even put forward that tec[h]nology is not the unique agent which determines ed[u]cation, nor is it the discipline mathematics. An inte[r]play between technology (e.g.: computers and so[ft]ware), the organisation of work, the discipli[ne] mathematics, and education takes place with mathem[a]tics education as an active part in this game. The re[]marks gave rise to a lively discussion: if and to wh[at] extent mathematics education should listen to what i[n]dustry describes as their mathematical needs. No co[n]sensus was reached, but differences in short-term an[d] long-term answers were discernable. R. STRAESSER al[so] alluded to different situations in industrialized cou[n]tries and rural, developing countries but this was no[t] commented on in the discussion (cf. part. 2.10 of th[e] report).

2.8 WG 1: Influence on mathematics as a science

The working group was given about 10 hours (two an[d] a half afternoon) to discuss and comment the influen[ce] of computers on mathematics. After a common discu[s]sion, the group of about 10–15 persons split into sub[]groups. It was agreed that the subgroups (sometim[es] only one person) should comment on the following to[]pics: new and revived areas of mathematical researc[h], proof, experimentation in mathematics, iterati[ve] methods, algorithms, symbolic manipulation system[s], computers and mathematical communication.

The group was aware that this was a more or les[s] pragmatic classification. Similar reasons are respo[n]sible for the fact that other aspects like visualization, new areas of application of mathematics and the effe[ct] on the "mathematical sciences", acceptance of numer[i]cal instead of analytic solutions, social changes an[d] challenges to the mathematical community were no[t] covered by any subgroup, although they had bee[n] mentioned as important at the beginning of the WG discussion. Therefore these aspects are not treated i[n] the draft WG report at the end of the conference.

The group report does not claim to have give[n] a comprehensive survey, but hopes to provide som[e] hints by describing a collection of examples of th[e] computer's influence. We will attempt to summari[ze] some of the most important points discussed in th[e] subgroups, quoting from the draft report several time[s] without indication.

Topics discussed in the plenary sessions were taken into account by several subgroups. If they have already been described in 2.1 or other chapters, they will not be repeated here.

The subgroup on *new and revived areas of mathematical research* gives several examples to substantiate that the computer has, and will continue to have, a significant impact on the directions of mathematics research, on the way in which mathematicians carry out their research, and that computers will not only commonly be used to arrive at conjectures, but also to assist in finding proofs. The subgroup finds it important to underline the continuity in the development of mathematics, too. The lecture of DAVENPORT on algorithms in computer algebra (see 2.5) was very impressive in this sense because it not only exemplified a revival of interest in a long-dormant theory, the theory of integration in closed form (Liouville) but also the application of important results of abstract algebra.

The subgroup on *proof* argues for an acceptance of types of proofs in which computers examine all of a finite set of cases like in the 4-colour-theorem. Accuracy and reliability of the computations should not be an issue today. It is believed that the computer will not increase the number of false proofs, rather the contrary is expected. The "brute force" and "no insight" arguments against computer-aided proofs are rejected, because some important problems like finding large primes or factoring large integers intrinsically require such methods, and whilst it may be true that a computer proof may bring little insight its very existence may inspire people to find more elegant or illuminating proofs (see below, the example of ALPAK use). On the other hand, it is underlined that no amount of numerical evidence constitutes a proof of a theorem relating to an infinite set, although such experiments may initially suggest what is true and, equally important, what is not.

The subgroup on *experimentation in mathematics* argues from a slightly different perspective, pointing to uses of Monte Carlo methods for studying statistical techniques and for studying mathematical models in the sciences. A tentative generalization of experimental results may not be acceptable as a premise in mathematical deductions, nevertheless such mathematical experiments may yield enough "certainty" to influence practical decisions or to stimulate scientific research. The computer has led to a broader variety of "types of rationality" and, dependent on the particular subject matter domain, an adequate professional practice is already emerging.

The revival of interest in *iterative methods* was viewed as one important effect on mathematics. Iterative methods of direct practical value have been constructed and analyzed, but the use of computers has also led to significant theoretical advances in the study of functions which are iteratively defined, e.g. dynamical systems, the results of which are more indirectly related to applications.

The new important role of *algorithms* in mathematics, science and society was underlined but without going far beyond the description already found in the ICMI background paper ([1]).

The subgroup on *symbolic manipulation systems* gives examples to illustrate that the significance of such systems is deeper than relieving mathematicians of a great deal of drudgery and encouraging them to attack problems which hitherto looked too intractable. In the plenary session H. O. POLLAK reported on one of the first uses of a symbolic package (ALPAK) at Bell Labs around 1960: ALPAK was used to carry out some very tedious algebraic manipulation involving 1.200 terms to find the second moment in a problem in queueing theory, which was of importance to Bell Labs. When the second moment was finally found, it reduced to just three terms, after which a shortened mathematical derivation was obtained and a general theory developed. So, starting with "brute force" may lead to significant theoretical advances. The theoretical significance was also underlined by drawing attention to the fact that recent symbolic software solves problems of which G. H. HARDY said in 1916 that there is reason to believe that no such solution method can be given.

The subgroup on *computers and mathematical communication* describes experiences and possibilities of using modern electronic networks (e.g. ARPA) and information retrieval systems to improve mathematical communication and research.

In summary, the WG report describes several informative examples and typical influences on mathematics as a science. It is also written with a view towards the mathematical community and addresses itself not only to mathematics educators and mathematics education. Just because the report does not claim to lay the foundation for a "new math in the computer age" which should be put into schools in some direct way, it is a valuable document which will also broaden the perspectives on reasonable computer uses in mathematics teaching.

2.9 WG 2: The impact of computers and computer science on the mathematics curriculum

According to the division of tasks given by the steering committee and the discussion document, the term "curriculum" for the working group's task should be taken in a very narrow sense, not including, for instance, questions of "teaching and learning" (methods) which were to be discussed in WG 3. So the discussions and papers of some subgroups concentrated on questions of content which was reported normally in the form of (commented) chapter headings of text books or lecture notes.

This narrowing was not shared by two rather big subgroups, namely that on calculus/analysis and that on exploration and discovery. The calculus group explicitely didn't want their subject to be artificially divided into content and methods, and the other group wanted to start work early, since some persons had to leave on Tuesday evening, and WG 3 started on Wednesday. Because of these movements, the title of the preliminary report of WG 2 was changed afterwards to "The impact of computers and computer science on the mathematics curriculum and the methodology of teaching mathematics".

The work of WG 2 was mainly in the subgroups, with little exchange of ideas between these (besides the papers in [2] and the discussions in the plenary sessions). So we shall just describe the positions named in the preliminary report of WG 2 (except for calculus, where we have more insight in the working-mode of the subgroup).

The preliminary report of WG 2 has the following structure:

I. The common mathematical needs of students in mathematics, science and engineering
    A. Preparation for university mathematics
    B. The university mathematics curriculum

II. A discussion of particular curriculum areas on which computers and informatics have an impact
    A. Discrete mathematics courses
    B. Calculus in the computer age
        1. The role and relevance of calculus
        2. The content of calculus courses
        3. Computers for learning and teaching
    C. Logic for mathematics and computer scientists

III. Exploration and discovery in mathematics

Appendix A. Symbolic mathematical systems

Appendix B. Exploratory data analysis

In I A the traditional topics: algebra, geometry, trigonometry, and calculus are shortly discussed, emphasizing a certain shift from manipulative skills to understanding. Probability and discrete mathematics arise as new contents but have to be pushed carefully since extensive teacher training is needed.

In I B the problems of the demands of computer science students are discussed, especially the question of continuous and/or discrete mathematics courses in the first semesters and the impact on students of other disciplines.

In II A the presentations and discussions of the plenary session on Tuesday morning (cf. 2.2 of this report) are reflected and a discrete mathematics syllabus is presented, naming sets, logic, induction and recursivity, combinatorics, difference equations, graphs, discrete probability, and as optional topics matrix algebra, number systems, algebraic structures, finite state machines. The necessity is stressed to build a coherent course instead of a potpourri.

In a common session the subgroup on calculus discussed the field and came to a certain global consensus on the problems and directions to be mentioned in the report. Afterward they split up into 4 writing teams whose results were commonly examined and approved. By this procedure, the subgroup's paper became considerably less detailed but acquired a certain coherence.

II B 1 names the calculus as one of humankind's greatest intellectual achievements, its realm of applications, the necessity to understand the interplay between discrete and continuous methods in the application context and the impact of powerful numerical, graphical and symbolic mathematical software in the knowledge needed to handle calculus models. II B 2 describes some consequences on the emphasis placed on different topics in the calculus courses and stresses the need to include also some discrete calculus and to study the necessary transitions as well. II B 3 discusses the new possibilities for discovery and exploration in the calculus opened up by some software (but much more of this is needed), the benefits of students writing their own short programs, and the changing of the order of topics within courses since preliminary work with concepts by means of special software seems possible and motivating for their formal elaboration. Consequences for the availability of resources in- and outside of the classrooms are named, especially the need to train teachers.

In II C a specific view on the logic courses for computer science students is developed which might also be of interest for other disciplines, especially for st dents beginning mathematics at universities.

In III it is argued for the benefits of exploration an discovery in learning mathematics; by active learnin the student may better grasp the idea that mathematic is an integral part of the human culture, its beauty, th aesthetic experience of the "aha", and the usefulnes universality and unity of mathematics. Computer tech nology may assist mathematical exploration and dis covery in a variety of ways, for instance, by visualiz tion of two- and three-dimensional objects or of th geometrical loci of (sets of) equations, by graphical an numerical explorations of definitions, through new an interesting geometries (turtle, flatland), explorator data analysis, approximation of functions, experiment ing with formulas (computer algebra), designing an analysing algorithms. Students should be confronte with simpler tasks in the beginning in order to giv them their own freedom to explore. Once again, th need for extensive teacher training for being able t handle these instructional ideas is stressed.

2.10 WG 3: Computers and computer science as an aid f teaching and learning mathematics

The working group had been alotted 10 hours (tw and a half afternoon) to produce comments on part 3 o the discussion document (cf. [1]). In order to cope wit this broad task, the group of approximately 30 person briefly discussed the assumption that the didacti triangle of pupils, teacher and knowledge is now sup plemented by the computer, forming a new quadran gular configuration. Then the group split up into sever subgroups which used the two afternoons to write u comments on topics agreed on beforehand, while th last two hours served to have these comments discus sed in a very cursory way.

The seven subgroups were to comment on: attitude towards mathematics / interaction between pupil teachers, knowledge and machines / treatment of spe cial areas / software development / cultural, social an economic conditions of educational computer use / as sessment / teacher training. As it is virtually impos sible to report on all the seven subgroups, we hav attempted to sort out the most important features o the comments produced by the subgroups, judging b a preliminary version of what is to be published in th proceedings of the symposium (cf. part 3 of the report) Some phrases below are direct quotations from thi version, even if not specified as such.

With respect to *knowledge* (i.e. mathematics) th group opted for a balance between experimenta mathematics and formal mathematics in which wel chosen illustrations from a computer should help buil intuitions. The value of computer sessions depend largely upon follow-up activities. A balance should b reached in the question of black boxes too: it seem impossible to explain all the scientific knowledge in corporated in sophisticated software packages befor use. Students should gradually acquire part of thi built-in knowledge. Even the selfevaluation and as sessment procedures, which may be easier with com puter use, should respect a balanced view of mathema tics, including features difficult to test with multipl choice procedures.

With respect to *computers and software*, three types o software were identified:

1. Sophisticated (e.g. muMath) systems providing solutions to specific mathematical problems and developed by commercial companies. They have been designed without pedagogical goals in mind, while interest in their use as pedagogical tools is growing. Little is known about their use and consequences for learning and teaching.
2. Software packages suitable for use on microcomputers are often very demanding in pedagogical design, but not easily obtained and not self-supporting in commercial terms. These packages attempt to aid the mathematical development of the students. Again little is understood about their effects on teaching and learning.
3. General purpose programming languages which may be used as tools to foster student's mathematical development. Two major requests were raised in this context: a good channel of communication on educational software ("teachware") must be established and structured, fundamental research is urgently needed on educational use of computers and software in order to proceed from the exchange of personal opinions to knowledge and recommendations relying on research findings.

With respect to the *pupil/student* some optimistic views were brought forward: Computer use can activate and motivate students and may lead to increased self-confidence and self-paced learning and self-assessment without fear. The computer may enable teaching mixed ability groups by adjusting the difficulties of individual tasks according to the abilities of the individual student.

All these possibilities depend on *teachers* willing and able to introduce new patterns of classroom interaction. Instead of the format "lecture – examples/homework – examination" we may have "project work – interaction between student, machine and teacher – assessment". This optimistic outlook was complemented by the experience referred to above in part 2.3 of the report (cf. BURKHARDT and SARGENT in [3]) which identifies major impediments of this development.

With respect to *cultural and economic factors*, a lively discussion on the special situation of developing countries can be reported. Some participants thought of computer use as a major help in coping with the lack of teachers in developing countries, provided the development of high quality software is based on good learning models and respects the cultural values and practices of the country in question. Other participants pointed out a multitude of problems computer use implies in developing countries e.g. the lack of electricity in remote areas, the absence of software adapted to non-industrialized countries and financial problems.

There was one topic running through most contributions in this working group, that is the need for structured, fundamental research to overcome the stage of distribution of opinions. Together with development activities, pre-service and in-service teacher training, and keeping in mind that problems identified above are international in nature and should be attacked on an international (funding!) basis, research and development in mathematics education may transform the computer and software into a valuable help for teaching and learning mathematics.

3. Outcomes of the symposium

About the symposium, two publications will be available: The "Document de Travail" ([2]) was updated and enlarged, thus forming a valuable collection of "Supporting Papers" ([3]) to the conference. This publication includes the ICMI-discussion-document ([1]) and 49 papers on general matters, mathematics education and curricula, experience with educational use of computers, symbolic systems and algebra, stochastics, discrete and continuous mathematics and visualizations (these are the headlines of the seven chapters). A list of participants of the symposium rounds off this informative document on "The influence of computers and informatics on mathematics and its teaching". It can be obtained from the IREM of Strasbourg (details of how to order [3] are given in part 5 of this report).

The proceedings of the symposium [4] are published by Cambridge University Press. The proceedings contain the discussion document, papers from each of the three working groups (cf. part 2.8 to 2.10 of this report) and invited papers.

4. Final remarks

Looking back at the symposium, we should like to identify the following major issues:

Judging from the procedure in the FR Germany, it would have been helpful if a *broader publication of the discussion document* had occured. In the FR Germany, it was more or less by chance that interested persons knew about the document. A more elaborated policy of publication might have led to better use of the existing professional knowledge of the influence computers have and will have on mathematics and its teaching.

Appropriate *software* – "amateur" or "professional" – seems to be a problem of the influence computers may have on mathematics and its teaching. The symposium presented some very interesting examples of software for educational and professional use, but the symposium also showed that there is no balance of supply and demand in software. There are deficits on either side.

It is virtually impossible to "open all *black boxes*" incorporated in the hardware and software used in mathematics and its teaching. The only way to treat this problem is to decide which amount of "greyness" is appropriate in a given use of computer technology. Decisions taken cannot rely upon scientific knowledge about the "black box-problem".

The use of computer technology obviously influences the *relation between rigour,* formalization and logic on one side, *and experiments,* intuition and visualization on the other. At the symposium, conflicting arguments and view points were brought forward and no consensus was reached.

Discussions at the symposium gradually shifted from topics related to computer use in the discipline of mathematics at the beginning of the symposium to educational use in mathematics learning during the last session of the symposium. The introductory remarks of ICMI-president J. P. KAHANE (France) and his conference-summary at the end of the symposium could very well illustrate this development. Discussing both subjects – *disciplinary as well as educational use of computers* in mathematics – in close connection proved workable and valuable. The International Commission on Mathematics Instruction (ICMI) thus started its study series with an interesting, important and stimulating experience.

One issue ran through the whole symposium and should be emphasized as most important: the symposium clearly showed that the "influence of computers and informatics on mathematics and its teaching" can be described and in part foreseen on the basis of (personal) experience and informed hypothesis. What is badly needed is a transition to structured, fundamental *research* on the theme of the symposium – together with development activities and teacher training.

5. Related literature

[1] ICMI 84: ICMI (Ed.): The influence of Computers and Informatics on Mathematics and its Teaching. – In: L'Enseignement Mathematique 30 (1984), p. 159–172. Partly reproduced in: ZDM 16 (1984) 6, p. 214

[2] DocTrav: Commission Internationale de l'Enseignement Mathematique (Ed.): The Influence of Computers and Informatics on Mathematics and its Teaching. Document de Travail (with supplement). – Strasbourg: IREM, 1985

[3] SupPap: Commission Internationale de l'Enseignement Mathematique (Ed.): The Influence of Computers and Informatics on Mathematics and its Teaching – Supporting Papers. – Strasbourg: IREM, 1985. SupPap can be ordered by payment of 100,- FF to "M. l'Agent Comptable de L'U.L.P.", C.C.P. Strasbourg 550644. The payment must contain a clear identification of SupPap.

[4] HOWSON, A. G.; KAHANE, J. P. (eds.): The Influence of Computers and Informatics and its Teaching. Proceedings of the Symposium. – Cambridge University Press.

TURBO PASCAL SOFTWARE

The French National Sub-Commission of ICMI decided at an international meeting which it mounted in January 1986 as a follow-up to the Strasbourg 'Computer' seminar, to create an annotated catalogue of available Software in Turbo Pascal for use in the teaching of mathematics.

Those wishing to submit descriptions or to receive further information should contact Robert Rolland, CIRM/IREM, Faculté des Sciences de Luminy, Case 901, 13288 Marseille Cédex 9, France.

ICMI AND SOUTH AFRICA

At the recent meeting of the ICMI EC a discussion took place on the subject of South Africa and the participation of South Africans in open meetings mounted under ICMI auspices.

Here it must be clearly stated that ICMI is a sub-commission of the International Mathematical Union, one of the members of the International Council of Scientific Unions (ICSU). This last body, ICSU, has a clear, unambiguous policy concerning 'the free circulation of scientists': "ICSU maintains that scientists from all parts of the world have the right to participate in its activities without regard to race, religion, political philosophy, ethnic origin, citizenship, language or sex" (ICSU, 1985).

It was the unanimous decision of those of the ICMI's EC present at the meeting that ICMI should abide by ICSU rules and that it wished affiliated study groups to keep to them also.

The meeting was also unanimous in condemning the 'Apartheid' policies of the present South African regime and in deploring the riots and killings there, whatever their immediate cause. Although abhorrence was shared, there was no general consensus on how a scientific body, such as ICMI, should react to this particular situation. In order to carry forward the debate and to bring conflicting views into the open, it was, therefore, agreed that members of the EC should have the opportunity in this Bulletin to put forward their personal opinions.

Perhaps at this point one might with profit quote from an article by Denis Galligan, Professor of Law at Southampton University. Here a ban on South African participation was imposed by the local organising committee of the World Archaeological Congress which in its turn led to the International Union of Prehistoric and Protohistoric Sciences disassociating itself from the Southampton Congress and mounting an alternative Congress in Mainz, West Germany:

"Issues of practical ethics rarely offer easy solutions. What appear at a distance to be clear and stable principles are likely on closer examination to dissolve into a mass of shifting parts which are elusive, enigmatic, and equivocal. And even when a principle can be held steady, its application to a real set of facts is likely to set off another train of complexity and ambiguity; evidence has to be got and interpreted, means have to be evaluated, and judgments have to be made. The debate over participation in the Congress by South Africans has all these hallmarks so that amongst men and women apparently of good will, the vast majority, if not all, of whom condemn apartheid, very different and irreconcilable conclusions are reached. There need not, of course, be any "right" solution to such matters, but clearly some conclusions are better than others and not all the arguments advanced are equally compelling."

AN ARGUMENT IN FAVOUR OF KEEPING THE PRESENT ICSU RULES

Attempts to ban South Africans would appear to rest on the following premises:

(a) Apartheid is so repugnant in theory and practice that strenuous efforts should be made to remove it;

(b) Pressure from outside is needed to achieve that aim;

(c) Banning South Africans from attending scientific congresses is an effective and, in the circumstances, justifiable form of pressure.

I should not wish to dispute (a) - my experience of apartheid in action was confined to 24 hours or so spent in transit in South Africa, but even that short time was sufficient to convince me that I had no wish to return there until the system is reformed.

(b) would also seem unobjectionable. However, there will be a vast difference in the effects of different types of pressure.

The crunch then comes with (c), the use of congress participation as a means for effecting political change in another country.

First, is it likely to prove effective? Not, surely, on any past evidence. Yet, I do not believe any action should be dismissed solely on the grounds of apparent futility: a point is being made, valuable encouragement may be being given to those working on longer-term, but more effective initiatives.

Yet, there is another side to effectiveness. When one uses a weapon, one must also consider who, in addition to those at whom it is directed, might be injured when it goes off.

Here, I have considerable misgivings. The use of Congresses as political weapons is not new. Immediately after the First World War the Conseil International des Recherches (ICSU's predecessor) pursued such a policy. German and Austrian mathematicians were not invited to the International Congresses of Mathematicians held at Strasbourg (1920) and Toronto (1924). The Union Mathématique International 'interdicted the dispatch of invitations to Germany (which had not yet joined the Conseil) even when scheduling the next congress to be held in Bologna, in September, 1928. The Italian organizers of the congress, however, ... invited mathematical societies from all countries including Germany and Austria' (Menger, 1979). It was at the Bologna conference that ICMI (inactive since 1918) was reconstituted with the German, Lietzmann, on the Executive Committee. Since that time it has been the policy to attempt to dissociate Congresses and politics, and ICSU, in particular, has always tried to pursue a policy of universality.

Now, clearly, if we are to break that policy then,

(a) the arguments for doing so must be strong, fair and unequivocal,

(b) the effects of making that break must be fully and carefully considered.

So far as (a) is concerned, the arguments are strong. But is the situation in South Africa any worse then in X (which has a fifteen year or more record of tribal genocide), in Y (in which religious minorities have been put to death on a frightening scale) or in Z (where a corrupt government has repressed its people through force)? Should not we ban participants from these countries also?

Whatever arguments we, as individuals, might give to this last question, there is no doubt in my mind that once it is accepted that Congresses may be used for political ends then everyone will join in. At the moment the ICSU rules enable scientists to obtain visas to visit countries which would normally refuse them entry. Their aim is to protect scientists (and science) against the actions of governments. If scientists once decide that certain countries are 'good' and others 'bad', why should not governments be allowed to do so also? If one makes an exception of South Africa, why not, say, Israel? Australia in 1982 barred two Soviet scientists from attending the International Congress of Biochemistry because of Afghanistan. (ICSU rules were then rigidly applied for the Physiological Sciences meeting in Sydney in 1983 and for ICME5 in Adelaide in 1984.) At present, so my Australasian friends tell me, feeling is running high against the French because of their nuclear testing policy (and Greenpeace). Why not then made a political gesture by banning the French from any Australasian meeting? No USA Federal funds were made available for Americans to visit the Warsaw ICM. Had the 1982 ICM been in the USA, would Poles have been banned from attending that?

What are we doing in scientific terms when we ban a South African from attending an international meeting? We are barring all parties from 'learning'. Now if this is a case of 'learning of an experiment in teaching decimals', the cost may not be too high. Suppose, though, that a South African had a proof of the Riemann Hypothesis or the Poincaré Conjecture. Would we refuse to listen to him/her? Two unions associated with ICSU are on Immunology and Nutritional Sciences. Would we wish to deny information to South Africans on these subjects, or to refuse to learn new findings and techniques from them? Do we then have to have different rules for different ICSU unions?

How do we persuade South Africans (and others) that there are better ways of doing things, if we confine them to their laager? If our case is so strong, why do we reject putting it to South Africans in person?

However, let me emphasise this is not a plea for 'open access' come what may - that Mengele should have been allowed to participate in Medical Congresses. The ICSU rules do not forbid the banning of individuals when it is known that they have acted in a way that outrages moral decency. They do forbid bans on the ground of citizenship alone.

The matter is not easy, but on balance I fear we have much more to lose than to gain from banning South Africans. It could cause local problems with anti-apartheid groups, but refusing to fight against local pressures is precisely the fault we laid at the door of the 1930s Germans and, now, at white South Africans. However, resisting such pressure groups is all the more difficult when one actually agrees with their goals, if not always with the means by which they pursue them.

A.G. Howson.

References

ICSU, Advice to Organizers of International Scientific Meetings, Paris, 1985.

Menger, K., Selected Papers in Logic and Foundations, Didactics, Economics, Reidel, Dordrecht, 1979, pp. 248-9.

AN ARGUMENT IN FAVOUR OF CHANGING THE PRESENT ICSU RULES

Our Co-responsibility for the South African Development

1. Not a political issue, but a question of human rights

During the last decades, it has become internationally recognized that the oppression of the black population by the government of the Republic of South Africa is rapidly increasing. This intensification of the suppression has been demonstrated not only by the long line of governmental terror acts against the black people within the Republic and adjacent countries, but, moreover by official governmental statements. Thus, the government not only defends its right to maintain the imperialistic philosophy of an all embracing white superiority, but it *openly* builds its policy and the whole existence of the Republic upon an exploitation, a suppression, and a persecution of the huge black majority which is based upon *racism as a system*.

I have often met the argument that the apartheid issue is of a purely *political* nature. I disagree. It is in my opinion rather of a *moral*, or for me, preferably, of an *ethical* type. Similarly, I see the hope for mankind (in the perspective of the nuclear dangers) in the fact that the question of human rights is being discussed and acted upon at a level, which is seen to be above the political one, and above the level of the various religions. In fact, agreement on, and respect for, human rights rest on an acceptance of a level of general (or universal) ethics. And human rights are becoming an influential factor, because a large majority of mankind hopes that fundamental aspects of being human can be agreed upon regardless of individual motives, political goals, socio-cultural traditions, and religious beliefs.

2. The personal and the social dimensions of the issue

In situations where serious decisions of a moral or ethical nature must be taken, the individual may have to supplement his/her general attitudes and viewpoints by consciously recognized personal border lines. The individual may - in order to maintain self-respect and balance - have to stipulate "limits": "This is as far as *I* can go. *Here,* I say no!" I have reached my personal limit with respect to the apartheid issue. I cannot accept the rationale, neither show acquiescence with the actions, of the South African apartheid system; and consequently, I must react against these actions.

In fact, to make such a personal decision - and to attempt acting accordingly is the easy part of the problem field under debate. The difficult, ethical problem is, whether general decisions may be made according to which groups of individuals - brought together for quite different, specific purposes - are recommended or even requested to react to the apartheid issue in similar and codified ways.

The tensions seem to arise from conflicts between such "easy" personal decisions and the individual's specific obligations and loyalties related to the network of social groups which constitutes the framework of life and work of man in society: the obligations and loyalties related to his peer groups, to his social strata, to his friends, etc. Here, again, we are back at the question of the *universality of human rights*. Are we - in the present case - dealing with an issue of such an overall nature for mankind that it justifies the drawing of general conclusions which may disrupt existing connections between individuals, and between individuals and the groups with which they are associated, such as their professional groups. My answer - which must again be a personal value judgement - is affirmative.

In the present context the "ICSU rules" become a factor of high influence. Thus, on the one hand, ICSU supports the individual and the professional groups by providing background for certain types of codified steps. But, on the other hand, exactly this support or protection serves also to establish limitations in the degree of freedom for decisions on the issue in question.

3. The ICSU rules about international meetings

ICSU is an international non-governmental organization composed of 20 international Scientific Unions and more than 70 academies of science and councils of research. Its principal objective is to encourage international scientific activity for the benefit of mankind.

A standing committee on the Free Circulation of Scientists was established by ICSU in 1963. One task of this Committee is to collect, document and analyse cases where bona fide scientists have been seriously restricted in the pursuit of scientific research or have been prevented from communicating with fellow scientists, and, in continuation, to assist the Executive Board of ICSU in taking appropriate steps whenever such restrictions have occurred.

The Committee has in its "Advice to Organizers of Scientific Meetings" provided advice to the organizers of international meetings "on a number of measures which will help to avoid difficulties". For the present purpose, the ICSU "rules" about international meetings may be feasibly represented by these quotations from the above mentioned publication.

> Statute 5 of ICSU: In pursuing these objectives [the objectives of the council] ICSU shall observe the basic policy of non-discrimination and affirm the rights of scientists throughout the world to adhere to or associate with international scientific activity without regard to race, religion, political philosophy, ethnic origin, citizenship, language or sex. ICSU shall recognize and respect independence of the internal scientific planning of its national Members.

> Organization of Meetings, paragraph 1: The organizers of meeting should always keep in mind that bona fide scientists should not be excluded from participating in international scientific meetings because of "... race, religion, political philosophy, ethnic origin, citizenship, language or sex".

4. The Paris meeting of the ICMI EC

Valuefree science does not exist. But access to the most recent results and information about developments within the field is vital for scientific quality in all domains and in all countries. Accordingly, ICSU's efforts to ensure freedom to participate in scientific meetings throughout the world have been of basic importance for scientific development during recent decades.

Thus, at the Paris meeting of the ICMI Executive Committee my conflict was the following. On the one hand, I had, clearly, my rights to maintain and declare my personal opinion against apartheid, and to recommend actions accordingly. On the other hand, however, I could not support any breaking of the ICSU rules for international meetings. Formally, because I am a long standing member of the Executive Committee of ICMI - which is a commission established by the International Mathematical Union, a member of ICSU. But also, because my above mentioned positive evaluation of the ICSU rules has been supported by my experiences in the period 1978-82 as the representative of IMU on the ICSU Committee on the Teaching of Science.

Accordingly, but with regret, because I wanted to react strongly - and in clearly noticeable ways - against the apartheid system, I had to support the proposal that ICMI and its affiliated study groups should comply with the ICSU rules. At the same time, I accepted with great appreciation this offer to clarify my position. Since then, I have tried to decide the details of the common steps in the international cooperation which I want to recommend, and also to decide under which circumstances I would find it feasible that ICSU makes an amendment to its rules according to which an exception is made for the Republic of South Africa - on the ground of its flagrant violations of the primary human rights principle of anti-apartheid - to be in force until the apartheid system has been fully abandoned by the government.

5. The necessity of sanctions

The main issue of the international debate about the Republic of South Africa is not apartheid. For years, now, general agreement has been expressed by governments all over the political spectrum to the effect that the apartheid system should and must be abandoned. The major issue for the external alignment is clearly, whether the system should be brought to fall by means of binding economic sanctions instituted by the United Nations against the Republic. And here the emptiness of such protests against apartheid has been demonstrated in the U.N. Security Council debates, where such sanctions have been vetoed repeatedly by declared opponents of the apartheid system, and likewise in the European Community Council debates which have resulted only in the weakest imaginable sanctions.

The exploitation of the resources of the colonies by the European colonial powers has been terminated during the years since World War II. But as an anachronism, the Republic of South Africa protracts the worst forms of exploitation by means of brutal racism developed into a legalised system. The corresponding steps and actions of the Republic must be seen as attempts to oppose and obstruct the first mentioned development of appeasement between people of different "race, religion, political philosophy, ethnic origin", a development which was too long delayed - at the cost of several generations of the original population. *Any further delay in total abandonment of apartheid on the part of the government of R.S.A. must be recognized and acted upon as a crime against mankind*. Such a delay will cause not only further bloodshed and brutality between white and black, and between opposing factions of blacks, but may cause the impossibility of arriving at some negotiated solutions to the catastrophic situation of Southern Africa.

The "cosmetic" reforms of the Botha government represent such a delay. And the same counts for the government's recent reaction to the Commonwealth proposal. The visiting delegation had proposed that the government should recognize ANC, free Nelson Mandela and other African leaders, and negotiate with these persons. In return, the fight against the government should be called off. The governments's flat rejection was followed by the bombing of three member-states of the Commonwealth, and the delegation left in anger.

It seems to be high time to follow the advice of the black leaders (such as Bishop Tutu) to use the strong means of binding economic sanctions; high time for the opponents of such sanctions to stop their repeated futile attempts at explaining away their lack of consequence. The serious economic difficulties of the government demands a positive economic development and an increased employment rate. When and if binding sanctions are applied, the apartheid system *must* be terminated to obtain the external support needed.

Different socio-cultural contexts provide different background for the personal value judgements concerning the South African development. In Scandinavia, the escalation of the oppression of the black population caused during 1985 a growing agreement that the limit had been reached and that strong measures had to be taken against all types of cooperation with subjects of the Republic of South Africa.

Thus, last October the Nordic Foreign ministers agreed that the rules under which subjects of R.S.A. may obtain visas to the Nordic countries should be made more rigorous. The new rules now in force in Denmark prohibit the granting of a visa to a subject of R.S.A. for the purpose of participating in events of sport, culture and science, and in conferences, congresses, exhibitions, etc. At the same time, the Danish Sports Federation requested, after eight years of hesitation, that all member organizations should refrain from participating in tournaments and similar events in R.S.A.

In this period, the Danish National Union of General Workers established a boycott of the unloading of all ships carrying coal from R.S.A. to the Danish Power Stations. The Danish Labour Court ruled in due time that the boycott was illegal, although the Union could refer to the fact that a confirmed majority of the Danish Parliament was in favour of prohibiting all commerce with R.S.A. In fact, a bill to that effect was passed by the Parliament in late May this year.

Were the above events at the turn of the year of my concern? Are similar events in other countries of interest and concern for mathematics educators and other intellectual workers? For me, the answer is clear. The background and motivation for the actions of the working men and women are certainly not linked to the quality of the coal from R.S.A. Their motive is that apartheid must be terminated. Neither do I need to qualify my request for sanctions - or for other steps - by any other reason. And, certainly, the responsibility for the costs of boycotts and sanctions must rest on the intellectual groups just as much as on any other group of society.

6. Some personal questions and proposals

The Nordic visa rules are in open conflict with the "ICSU rules" described in 3 above. What happens when other countries join in such stronger measures against the upholder of the apartheid system? What happens, when the United Nations agree on binding economic sanctions? Would this not make it imperative for ICSU - as a Council supported by and closely cooperating with the U.N. organization Unesco - to make an amendment to its rules of the type indicated at the end of section 4 above? But would it not be more appropriate if ICSU - in the perspective of its Statute 5 - took steps at an earlier date to quell the apartheid system by a ruling of a temporary exception of R.S.A. from "the family of nations"? I am at the time of writing deliberating, whether I should make a formal recommendation to ICMI that the Commission takes steps to ask ICSU to consider the feasibility of such a ruling.

Meanwhile, let me conclude by mentioning some principles which I recommend to be observed commonly in international cooperation concerning our field, principles which I hereby propose be supported by ICMI. These steps represent a boycott of various types of "intellectual products" by subjects of the Republic of South Africa. This boycott is meant as a measure against the apartheid system of R.S.A., not as an action against the persons behind the products. It aims at demonstrating for the government that the apartheid system *must* be terminated. Only then will it again be acceptable by the international scientific communities that some specified functions in relation to international cooperation are conducted by subjects of the republic. As it will be observed, the functions in question are traditionally seen as reflecting honour and recognition not only on the contributor in question, but also on his/her country.

While waiting for decisions of ICSU concerning amendments to its "rules", no denial of the right of the subjects of the Republic of South Africa to participate in meetings should take place. However, such subjects should for the time being *not* be asked to act in the following professional roles during international meetings or in other international cooperative efforts: (1) as invited speakers; (2) as members of planning committees; (3) as chairpersons; (4) as discussion leaders; (5) as referees or reporters; (6) as editors; (7) as contributors to periodicals or books. Exceptions: Subjects of the Republic of South Africa who are recognized as opposers of apartheid.

10 June 1986 Bent Christiansen

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NOTE