

# **ICMI**

## **Bulletin**

of the  
**International Commission**  
on  
**Mathematical Instruction**

**No. 27**

**December 1989**

Secretariat  
Centre for Mathematics Education  
University of Southampton  
Southampton, SO9 5NH  
England



**The International Commission on Mathematical Instruction**

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**December 1989**

**Editors:** Keith Hirst and Geoffrey Howson  
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University of Southampton  
Southampton, S09 5NH  
England

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Mathematical Instruction  
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# ANNOUNCEMENT

The International Programme Committee appointed by ICMI  
announces a study conference on

## "Assessment in Mathematics Education and its Effects"

Calonge (Costa Brava), Spain. April 11-16, 1991

As one of its main outcomes, the study conference will contribute to a volume on this theme to be published in the series of ICMI Studies.

A discussion document written by the Programme Committee will provide further details about the aims, scope and issues of the study conference. This document will be published in ICMI's official journal, "l'Enseignement mathématique", and elsewhere in the spring 1990. However, at this stage the Programme Committee is interested in contacting individuals or groups to contribute papers to the study conference. A brief outline of the goals and structure of the study conference is presented below. Participation in the conference is by invitation only by the International Programme Committee, but those who submit a contribution for consideration are encouraged to apply for an invitation.

### Motivation for and goals of the study

Assessment and evaluation have always been subjects of debate in mathematics education. Although it is recognized that the purposes, roles, and functions of assessment and evaluation in mathematics education are viewed very differently in different educational systems and in different societies, these factors exist wherever mathematics education is institutionalized, regardless of the socio-economic, political, and cultural characteristics of the society in which it is embedded. Thus, for better and for worse, the importance and influence of assessment and evaluation cannot be denied for any educational system.

The terms "assessment" and "evaluation" are often used interchangeably without a clear difference in meaning between them. The Programme Committee suggests that "assessment" be used in connection with the performance of **students** as individuals or in groups, whereas "evaluation" should deal chiefly with instructional **programmes** and **curricula**. In the present study, the dominant emphasis will be on this definition of **assessment**. This does not imply that issues related to evaluation are to be left out of consideration in the study. However, the

evaluation of programmes and curricula will primarily be addressed as reflected in the assessment of students' performance. The appraisal of **teachers** is an important component in the evaluation of programmes, curricula, and educational systems. The Programme Committee holds the view that teachers' quality should not be judged solely on the basis of their students' performances.

The Programme Committee believes that many current assessment practices are counterproductive and products of outmoded educational traditions which fail to meet today's views about mathematics and societal needs. Thus, the goals of this conference are: to examine current assessment practices in many nations; and to identify examples and practices that enable assessment to become a positive influence on instruction.

### **Problems and issues**

The problems and issues to be addressed in the study conference concentrate on the various purposes, roles, and functions of assessment of students' mathematical performances. In particular, to contrast practices across countries, the following fundamental questions about assessment procedures and their uses need to be addressed:

- \* What is the historical development in the philosophy and the practice of assessment and evaluation?
- \* For what purposes is information about students' mathematical performances being gathered? (To help teachers make instructional decisions? To assist students in monitoring and controlling their own learning process? To select or place students? To evaluate the effects of new programmes?..)
- \* What are the "units" on which information is being gathered? (The individual student, group, class, teacher, institution, programme, system...)
- \* For what kinds of mathematical tasks are students' performances being assessed (short technical exercises, long tasks, extended problems, portfolios, project reports)? And what kind of information is being gathered (examination of written items, of oral presentations or responses to oral questions; observation of performance)?

- \* Who gathers the information?
- \* How is information gathered, coded, recorded, and analyzed?
- \* When and how often is the information gathered?
- \* What kinds of decision or action are taken as a consequence of the information gathered?
- \* Are new procedures being developed/tried out?
- \* Are there conflicting views or interests between different quarters in the educational system in relation to assessment and evaluation (e.g., between government authorities and mathematics educators)?
- \* What are the important differences in the assessment practices of different countries?
- \* How "useful" are international performance comparisons, such as IAEP and IAE?

While these questions mainly serve descriptive purposes, the following ones focus on analyzing different modes of assessment with particular regard to their influences and effects:

- \* What are the theoretical and empirical foundations of current assessment procedures, and to what extent are these procedures valid, reliable, efficient...?
- \* What are the known influences of external assessment practices on mathematics instruction?
- \* Are there examples of assessment practices which positively influence instruction? What aspects should be maintained and encouraged?
- \* What are the examples of assessment practices which negatively influence instruction; for instance, by focusing the teaching on assessment and tests rather than on more general curriculum goals?
- \* How do different assessment modes influence the social environment in the classroom?
- \* In what respects are teachers good/bad judges of students' performance? And to what extent can they be trained to be good judges?

- \* How does the teacher's evaluative role conflict with his/her supportive role? This is related to the fact that in many countries, university professors are considered capable of internally assessing students justly, validly, accurately, etc., whereas school teachers are not; thus, external examinations are judged necessary. Is this sensible?
- \* It is widely recognized that most current assessment practices mainly deal with lower level skills. Many of these practices have a high degree of reliability but a low degree of curriculum validity. To what extent is it possible to devise assessment modes which have both? How well do we assess higher level abilities such as scientific debate, problem formulation and problem solving, modelling, application, etc.?
- \* What assessment modes are suitable in relation to different types of tasks, such as short technical exercises, long tasks, extended problems, project work, etc.?
- \* How can assessment be embedded harmoniously into instructional practices as an instrument to serve the needs of both teachers and students in everyday instruction?
- \* What are the main obstacles to devising and implementing innovations in assessment, and what can be done to overcome these obstacles?

### Structure of the study

To accommodate a thorough treatment of the issues listed above, the study will contain four sections.

1. A **descriptive** section:

In this section, the most important different conceptions and modes of assessment practiced in different countries/educational systems will be identified and described. Emphasis will be on "archetypes" rather than on peculiarities.

2. An **analytic** section:

This section will establish a framework for analyzing goals, functions, effects, consequences, limitations, possibilities, difficulties, problems, etc., related to



assessment of students' mathematical capabilities. By means of this framework, specific analyses regarding the issues listed above will be carried out.

Furthermore, important empirical or theoretical research contributions to the field will be presented.

3. **Presentation and discussion of innovative/experimental assessment cases:**

In many places around the world, very interesting innovative/experimental work on new modes of assessment and evaluation has been/is being done. This section will present and discuss a number of the most interesting examples. Special emphasis will be put on cases dealing with higher level capabilities.

4. **Statement section:**

This section will formulate policy statements and recommendations if appropriate. At the least, it will serve to identify important open issues as well as objects of future research and development.

The Programme Committee will solicit contributors to address specific questions, to present research contributions, and to share examples of innovative work at the conference. We invite suggestions on topics and names of potential contributors at this time. Also, comments regarding the format and structure of the conference will be welcome. As indicated, further details of the study conference will be published in l'Enseignement mathématiques and elsewhere in the spring 1990.

The International Programme Committee consists of:

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# THE ICFI SEMINAR ON "THE POPULARIZATION OF MATHEMATICS"

Leeds, September 1989

(The following report was written by Alan Bishop, the U.K. National Representative, for the U.K. Sub-commission of ICFI.)

This seminar was held at Leeds University, from 17-22 September 1989, and ran in parallel with the Pop Maths Roadshow. It was attended by more than 80 people from 20 countries. The programme was a combination of plenary presentations and working groups, and covered a wide range of issues concerned with popularization: mathematics as part of culture, the image of maths in society, philosophies of popularization, as well as various approaches to popularization such as TV and films, books and magazines, exhibitions, radio and public lectures, games, puzzles, maths trails, workshops, clubs and competitions. Examples were presented of all these approaches and there was considerable interest and discussion of their relative strengths and weaknesses. The Pop Maths Roadshow offered an excellent range of examples of all these approaches and was a considerable source of interest to participants.

There was some amusement among the English speakers when they discovered that French colleagues use the word 'vulgarisation' as the translation of popularization. This word, in French, does of course have the full range of meanings of popularization, unlike its English 'partner' and Professor Chris Zeeman was relieved to discover that his presentation on Catastrophe Theory was described by the ICFI President, J P Kahane from France, as "high vulgarisation"!

There was a general consensus that the process of popularization is at least as important as any particular product and the negotiations concerned with say the production of peak time TV shows or Maths trails demonstrated the importance of popularizing among those who control the media, for example. At the heart of popularization is the engagement of more people with mathematics, and any activity which did that at any level was important to consider. There were of course debates over whether 'proper' mathematics was being popularized, and whether any popularization was good popularization, with much consideration of the 'populace' being targeted. Minority groups and alienated cultures received some attention although I felt rather minimally. There was a

working group on Mathematics in Different Cultures which made some important and general recommendations, and it is to be hoped that they will emerge in the final document.

The proceedings will appear in due course in the ICMI series published by CUP and could provide everyone in mathematics education with a useful resource. There are some exciting ideas being tried around the world, and it was good to sample some of them. Lynn Steen described his work in writing readable accounts of the latest mathematical research, Virginia Thompson talked of the Family Math Project in California, and M. Lesgards astounded everyone with the financial support given to the new Cité des Sciences in Paris. However the image which will stay with me the longest is that of Australia's Governor General being late for a crucial engagement because he was discussing Fibonacci with a twelve year old girl on one of Dudley Blane's maths trails! We are certainly doing some things well in this country, but we could do more.

Alan J. Bishop

## **8th INTERNATIONAL CONGRESS ON MATHEMATICS EDUCATION, 1996**

It is the intention of the Executive Committee of ICMI to make a decision early in 1991 on the site for ICME8. Countries wishing to bid to act as host should submit a preliminary bid to the President and Secretary no later than May, 1990.

# MATHEMATICS AND COGNITION

Mathematics and Cognition, the fourth of the ICMI study series volumes, will be published by the Cambridge University Press early in 1990. The book is about 200 pages in length. It can be ordered from any bookshop. The ISBN numbers and approximate UK and US prices are

Hard cover	ISBN 0521 366089	£20	US\$35
Paperback	0521 367875	£8.50	US\$15

The editors are Jeremy Kilpatrick and Perla Neshet.

Only very few copies of the first two study series volumes, The Influence of Computers and Informatics on Mathematics and its Teaching (Churchhouse *et al*) and School Mathematics in the 1990s (Howson and Wilson),

remain in stock.

The third volume (still available) is

Mathematics as a Service Subject (Howson *et al*).

All are published by the Cambridge University Press.

The 'Computers' volume is still available in Japanese (Tutti-Mori Agency). '1990s' is available in Japanese (Tutti-Mori) and Spanish (Mestral Books, Barcelona); Arabic and Finnish versions are planned. A book based on the first three studies will be published in Chinese in 1990. An updated version of 'Computers' is planned.

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## **ICME 6 BUDAPEST**

Much of the success of ICME 6 was due to the tireless efforts of Tibor Nemetz. We are delighted to report that in appreciation of Tibor's work the Australian participants at ICME 6 have contributed a generous amount to assist Tibor to attend ICME 7, Quebec, 1992.

# NEW NATIONAL SUB-COMMISSION

A National Subcommission of ICMI has been formed in Mexico, with the following members:

## **Coordinators:**

Emilio Lluis  
Elisa Bonilla  
Elfriede Wenzelburger  
Eduardo Mancera

## **Members:**

Alicia Avila  
Carlos Bosch  
Alfinio Flores  
Elias Loyola  
Efrén Marmolejo  
Rubén Moreira  
Lilia del Riego  
Eduardo Zárate

The coordinators have proposed the following goals and tasks for the Mexican Subcommission:

1. To be a link between the Executive Committee of ICMI and Mexican institutions and groups interested in mathematics education.
2. To assume all the responsibilities as national representative.
3. To keep the mathematics education community in Mexico well informed about ICMI's effort and activities.
4. To be a link between groups affiliated to ICMI (PME, ISGHPM, IOWME) and their Mexican members.
5. To participate in the organisation of Interamerican and regional meetings of CIAEM (IACME)
6. To participate in the organisation of national and local meetings on mathematics education in Mexico.
7. To promote national surveys on mathematics education in Mexico and to seek financial support from Government or other international agencies.
8. To promote and organise a future ICME in Mexico.

# IMO NEWSLETTER 9

The XXX International Mathematical Olympiad was held in Braunschweig, West Germany, July 13 - 24. 291 students from 50 countries took part.

The XXXI IMO will be held in the People's Republic of China, July 9 - 19, 1990.

Future host countries for the IMO are: Sweden (1991), Turkey (1993), Canada (1995).

## ICMI PROJECT: past IMO Team Members

ICMI wishes to compile a list of all past IMO Team Members, with some information about their present work. All countries are urged to contribute to this work. Information sent to John Hersee by April 1st, 1990 will be translated if necessary to provide versions in English, French and Russian, and will be duplicated for distribution at the 1990 IMO.

## IMO: questions on new topics

Every year at the IMO attempts are made to widen the range of topics used for problems, but usually with no success. To try to make progress the IMO Site Committee has suggested that problems on one specific new topic should be generated during the next **two** years, looking forward to the 1991 IMO. If you have suggestions for problems based on algorithms, please send them to Professor Arthur Engel, who has agreed to collect them. It is hoped that progress can be reviewed at the 1990 IMO.

Professor A. Engel, Am Treutengraben 49, 6000 Frankfurt/M. 90, Federal Republic of Germany.

## Proposed Item Bank of Olympiad Problems

UNESCO has provided funding for the creation of an Item Bank of problems and solutions used in mathematics competitions.

Professor Valeri Vavilov of the USSR is collecting material for this Item Bank which is to be stored on disk on an IBM compatible computer.

Professor Vavilov hopes that Leaders of IMO teams will be able to help him to make the Item Bank truly representative of the wide range of questions used in national and international competitions throughout the world. He suggests that, in each country, questions and solutions used in mathematics competitions

should be collected and stored on disk on an IBM compatible computer. It is important that the items are all stored in the five UNESCO languages (English, French, German, Spanish, Russian). A copy of the disk could then be sent to him in Moscow. In return for help in providing material for the Item Bank, those who send a disk to Professor Vavilov will receive a copy of the whole Item Bank on disk.

UNESCO has provided funding for this work, but the idea could be extended in a number of ways. For example, some or all of the material could be published in book form. However, there are no UNESCO funds for such extension work. If the work is to be extended, some sponsorship will be required.

If you can help Professor Vavilov with this work, please write to him. You may be able to offer to prepare a disk for your own country. You may know someone else who would be prepared to produce a disk! You may have a suggestion for a possible sponsor so that the work can be extended!

If you can help, write to:

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119899 Moscow, USSR.

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John Hersee, Secretary, IMO Site Committee, 76 Pembroke Road,  
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# PSYCHOLOGY OF MATHEMATICS EDUCATION

## ICME-6 REPORT OF THE INTERNATIONAL GROUP PSYCHOLOGY OF MATHEMATICS EDUCATION

At Exeter in 1972 (ICME-2), Professor Efraim Fischbein instituted a working group bringing together people working in the area of the psychology of mathematics education. At Karlsruhe, in 1976 (ICME-3), this group became affiliated to the International Commission for Mathematics Instruction (ICMI).

The International Study Group for the Psychology of Mathematics Education (PME) has met each year since 1976 in many of the different countries from which its members derive.

The major goals of PME are:

- . to promote international contacts and exchange of scientific information;
- . to promote and stimulate interdisciplinary research with the cooperation of psychologists, mathematicians and mathematics teachers;
- . to further a deeper and more correct understanding of the psychological aspects of teaching and learning mathematics and the implications thereof.

There are several hundred members in the group. Membership is open to persons involved in active research to further the group's aims, or those professionally interested in the results of such research.

PME has a constitution which provides for the election of a President and an International Committee (see below the list of the present IC members and of the PME officers). As president of PME, Pearla Nesher (Israel) organised PME's contributions to ICME-6 with the assistance of the Hungarian coordinator László Mérö.

The following texts report briefly the activity of these sessions; a full presentation of these research activities will be given in the book to be published by Cambridge University Press during 1989 under the editorial responsibility of Pearla Nesher and Jeremy Kilpatrick.

**Epistemological questions concerning the learning and teaching of mathematics**

**Chair: G. Vergnaud (France)**

**Panel: G. Booker (Australia), A. Sfard (Israel), D. Wheeler (Canada)**

The idea of epistemology as the relationship between students' development of knowledge and problem solving, was introduced. An essential issue is that mathematics should be taught and learned as a progressive construction to face practical and theoretical problems. This thesis parallels the idea that, in the course of history, mathematical concepts, algorithms, reasonings and proofs have emerged that way. The concept of number was offered as a good illustration of such a long-term process.

The main message from epistemological studies is that the learning of important concepts cannot be made simple, smooth and error free, as traditional pedagogy hoped. Achieved knowledge is always in some sense an obstacle to the achievement of further knowledge. This is still the case even when the epistemological approach is adjusted to take account not only of the obstacles arising from mathematics itself, but also from the cultural, temporal and pedagogical contexts within which learning takes place. These contentions were illustrated with some examples drawn from the field of algebra.

The most important problem in mathematics education has to do with the implicit epistemologies of teachers and students. Textbooks rely heavily on implicit epistemologies, which have very little connection with the way students develop and grasp mathematical concepts and procedures. Mathematicians felt easy with a structural conception of mathematics, whereas students especially young students, have rather a procedural or operational conception of mathematics. Before regarding mathematical entities as objects, it is necessary to grasp their usefulness as tools to understand and progress. Examples such as the concepts of number and function have gradually become objects in the course of the history of mathematics.

There is often a gap between the teacher's presentation and the students' views of the same idea; it is probably the most important factor contributing to the ineffectiveness of mathematics education.



**It is the researcher's duty to explore the students' and the teachers' implicit epistemologies and the dangerous gap between the two. How do learners shift from a point of view to a new one? Under which conditions?**

**Epistemology of mathematics education necessarily relies upon epistemology of mathematics itself, even if it does not parallel it. The central question concerns the nature of mathematics: is mathematics a construction, or the discovery of preexisting entities? The answer to this question will have strong consequences upon education. The history and the epistemology of mathematics should be a standard component of teachers' training.**

**Psychological research on the learning of algebra**

**Chair: Carolyn Kieran (Canada)**

**Panel: Eugenio Filloy (Mexico), George Booker (Australia)**

**This session consisted of three presentations, followed by a discussion period involving audience participation. The first presentation summarized the major findings of algebra research by PME members over the last ten years. Particular attention was drawn to the difficulties that beginning algebra students experience with variables and the notational conventions of algebra, as well as with the structure of algebraic expressions and equations.**

**The second presentation focused on recent theoretical considerations in the field of algebra research. It was proposed that theoretical models need to include three components: algebraic teaching models, models of the cognitive processing involved in learning algebra, and formal competence models that simulate the performance of an ideal user of algebraic language.**

**The third presentation emphasised pedagogical implications of PME algebra research. It discussed, in particular, some of the studies that have aimed at building upon the arithmetic frame of reference that children bring with them to the study of algebra and at helping them bridge the gap between arithmetic and algebra.**

The contribution of psychological research to the learning of geometrical concepts

Chair: Rina Hershkowitz (Israel)

Panel: David Ben Haim (Israel), Claude Gaulin (Canada), Glenda Lappan (USA), Celia Hoyles (United Kingdom)

The two main "classic" aspects of teaching and learning geometry -- geometry as the science of space, and geometry as a deductive mathematical science -- were the implied background of the discussions. The specific topics discussed were the following.

There is a general agreement that spatial ability is important, not only for its own sake, but because the type of mental processes involved in visualisation are necessary for, and can transfer to, other areas of mathematics, especially to geometry in which visual elements are some of the "building blocks". In this spirit, three kinds of studies were discussed: (1) spatial ability itself; (2) the relations between spatial ability and other mathematical abilities; (3) the effect of instructional interventions with spatial tasks sequences on the learning of different areas in mathematics.

Processes and factors which have an effect on concept attainment were discussed. In order to clarify the nature and the place of this effect, a logical analysis of the relationships between different elements involved in the attainment of a geometrical concept was performed. Research examples of the attainment of basic geometrical concepts to more advanced concepts, like symmetry and similarity, were presented.

This topic was first linked to the Van Hiele theory and then research related to the notion of proof was discussed. This latter included (1) research which focuses on the process of proving: wrong arguments, all kinds of mistakes and illegal steps; (2) research which focuses on the conception of proof.

The reflection of geometry learning issues in the "mirror" of a computer learning environment was described, as well as additional dimensions which arise as a result of the use of this powerful medium in geometry teaching. Thus issues like conjectural thinking in discovery learning in geometry become more and more vivid.

## **Future perspectives of PME**

**Chair: Nicolas Balacheff (France)**

**Panel: Tom Romberg (USA), Christine Keitel (FRG), Pearla Neshet (Israel), Willi Dörfler (Austria)**

Learning and teaching depend to a great extent on socially available means for representing and communicating the intended knowledge. The latter even will be transformed if qualitatively new tools for knowledge representation are created. The most important new tool in this respect is the computer together with all other facilities of modern information technology. For psychological research, this opens vast and demanding new fields of research: how to use the computer as a learning and teaching aid? What are the effects of certain implementations on the cognitive development of the learner? What are epistemological changes due to the use of computers?

The different research perspective within PME has supported the development of a richness in the field of research work, and recently has gone beyond borrowed paradigms and restricted methodologies, in trying to construct more appropriate ones to the specific interests of PME. These shifts in the focus of PME which gave special broadening of research perspectives and research questions are: One shift is from the individual child to the pupil in the classroom. Another shift is from the search for universals in individual learning to specific characteristics in different social situations and respective change of them and the relation of learning and teaching in institutions. A third shift is from the psychology of mathematics education to the social psychology of mathematics education focusing on social aspects of the psychology of learning and teaching.

Research on cognitive aspects related to the learning of mathematics in a variety of countries has demonstrated that in the process of learning it is not sufficient to take into consideration merely mathematical factors. Psychological factors have emerged from extensive studies and enable us now to further categorise and sequentially order the learning of mathematical concepts. For example, we no longer speak of teaching addition and subtraction, but rather we distinguish among "combine", "change", and "compare" types of additive situations. Future research of PME

members should deal with other mathematical concepts learned at school and view them as cognitive-psychological entities. The learning of mathematics can take place only if such psychological and didactical considerations are taken into the learning plan.

## **ICMI - REGIONAL CONFERENCE**

The meeting planned for Beijing in September 1990 has been postponed and will now take place in September 1991.

## **BRITISH COUNCIL**

The 2nd Bratislava International Symposium on Mathematics Education will be held from 22 - 26 August. 1990.

(Dr. P. Bero, Katedra Matematiky, EF SVŠT, CSSR - 84219 Bratislava 29, Czechoslovakia)

## **PRIMARY MATHEMATICS:**

### **New Trends and Directions in the Curriculum**

**1 - 10 April 1990, Cambridge , UK**

The British Council is organising a conference on this theme, covering a number of major topic areas relating to mathematics 5-11, including problem-solving, language, new technology, cross curricular work, and raising issues relating to the needs of teachers and parents. Further details and application forms can be obtained from British Council Offices, or from Courses Department, The British Council, 65 Davies Street, London, W1Y 2AA, UK.

## **FIFTH SOUTH EAST ASIAN CONFERENCE ON MATHEMATICAL EDUCATION "ENCHANTMENT OF MATHEMATICS"**

**14 - 16 June 1990**

**Further details:**

**Dr. David Daniels  
Secretary, SEACME 5  
Universiti Brunei Darussalam  
Gadong, BSB 3186  
Brunei Darussalem.**

# INSTITUTIONS FOR MATHEMATICS EDUCATION

## 4. Undergraduate Mathematics Teaching Conference

Lecturers in mathematics departments in higher education are not generally renowned for their interest in teaching methods. Yet in the early 70's a new conference appeared to address precisely those issues - the teaching of mathematics to undergraduate students. It was set up by a group of eminent mathematicians who felt that they wanted to share with others their concerns about the nature of mathematics teaching in British universities. Given the traditional approach to mathematics lecturing, this was quite a radical step to take.

Since then the Undergraduate Mathematics Teaching Conference (UMTC) has become an annual event; 1990 will see the fifteenth in the series. Each year it is held at Nottingham University with the assistance of the Shell Centre for Mathematical Education. About fifty mathematics lecturers from British universities, polytechnics and colleges of higher education attend.

The format of the conference has proved so successful that it has remained relatively unchanged over the years. The meeting lasts three and a half days during the course of which participants meet in small working groups to address particular issues. Briefs for the groups are provided in advance outlining a particular task relating to a pertinent topic in the area of undergraduate mathematics education. By the penultimate day each group draws up a draft report which is passed on to a different group for comment. Taking into account the feedback and criticisms on the draft, a final report is produced which is published in the conference proceedings. Plenary sessions by invited speakers are built into the programme to stimulate discussions in the working groups and to provide information.

UMTC is invaluable because it facilitates exchange of ideas across the whole spectrum of higher education and it does so because the participants play an active part in the meeting. The process of producing the report, rather than the final report itself, is the important part of UMTC. The process provides the opportunity for delegates to think constructively about undergraduate mathematics teaching and related issues. The working groups provide the focus and the forum for discussion. Participants at the conference remark on the open atmosphere at the meetings - sharing ideas and seeking solutions to problems are the main concerns.

At each conference working groups discuss one of three different topics, perhaps united by a main theme. It is interesting that some of the burning issues resurface periodically. For example, whether and how to teach analysis to first year undergraduate students seems to be a perennial problem. Another issue that occurs in various guises is the role of technology in the teaching of mathematics. The technology has moved on - at one time the use of calculators would have been discussed, now it is the use of algebraic manipulation packages, but the issue is still there. Other recent topics discussed include the assessment of effectiveness in the teaching of mathematics, the role of the history of mathematics in the undergraduate curriculum and investigative work in undergraduate mathematics.

Perhaps the fact that perennials do occur is a sign that mathematics teaching in higher education is slow to respond to change. Nevertheless UMTC continues to challenge the traditional approach to teaching mathematics in higher education. In 1989 it joined forces with the London Mathematical Society to run *Mathematics 16-20* a meeting set up to consider the change in style and content of school mathematics syllabuses and its effect on mathematics courses in higher education in Britain. About 150 attended the five day meeting which included representatives from secondary schools, sixth form colleges and further as well as higher education.

The issues raised in *Mathematics 16-20* suggest that approaches to teaching mathematics in higher education need to alter radically in the near future. In a year's time the first candidates who took GCSE Mathematics will be admitted to higher education; these new undergraduates will be used to doing mathematics and not just learning about it. In addition mathematics degree programmes in higher education are not recruiting well. It is likely that mathematics courses will have to be redesigned to make them more attractive and to provide access for students with non-traditional entry qualifications, perhaps without the equivalent of an A level pass in mathematics. Mathematics in higher education in Britain may be about to face one of its biggest upheavals for some time and UMTC will have a significant role to play in dealing with the change.

Ruth Farwell,  
(Member of UMTC Organising Committee 1985-1990)  
St. Mary's College of Higher Education.

*The fifteenth UMTC will be held in September 1990. Details and availability of past proceedings may be obtained by writing to Judith Rowlands, Shell Centre for Mathematical Education, University of Nottingham, Nottingham NG7 2RD.*

# ICIAM 91

## Second International Conference on Industrial and Applied Mathematics July 8-12 1991, Washington, D.C. USA.

Details from

SIAM,  
3600 University City Science Center,  
Philadelphia,  
PA 19104-2688, USA.

## PME

The 14th annual Conference of the PME Group will be held at Oaxtepec Estado de Morelos, Mexico from 15 - 20 July, 1990.

## TME

The 4th TME Conference will be held at the same site from 2 - 8 July, 1990. Further information can be obtained from:

Professor D. H.G. Steiner  
Institut Für Didaktik der Mathematik  
Universität Bielfeld  
Postfach 8640  
Bielfeld 1  
WEST GERMANY



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