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The International Commission on Mathematical Instruction

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The International Commission on Mathematical Instruction

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Legend: IMU stands for *The International Mathematical Union*; ICSU stands for *The International Council of Scientific Unions*; CTS stands for *The Committee on the Teaching of Science* (of ICSU).

Next ICMI Executive Committee, 1995-1998

At its General Assembly, held in Luzern (Switzerland), in July/August 1994, the International Mathematical Union (IMU), which is the mother organisation of ICMI, appointed the next ICMI Executive Committee. The next EC which will serve from 1st January 1995 to 31 December 1995 is composed as follows:

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Carlos Vasco, Universidad Nacional de Colombia, Santafé de Bogota, DC (Colombia)
Dianzhou Zhang, East China Normal University, Shanghai (China)

Ex-Officio Members:
David Mumford, Harvard University (USA) (new President of IMU)
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Brief biographies of the new *members* are given below. The biographies are edited versions of autobiographies supplied by the biographees themselves.

Colette Laborde (France) is a graduate in mathematics from the Ecole Normale Supérieure. She obtained the "agrégation" in mathematics and a "doctorat ès sciences" in the didactics of mathematics at the science university of Grenoble (France) on language problems in mathematics teaching and learning. She is a full professor at the teacher institute of Université de Grenoble and is member of the research group DidaTech (didactics and cognitive technology in mathematics) at IMAG (Institute of Applied Mathematics and Computer Science in Grenoble). For quite a few years she has been involved in investigating the teaching and learning processes of geometry in computer based environments. In particular she is a contributor to the educational part of the development of the software *Cabri-géomètre*.

Since 1985 Colette Laborde has been the head of the doctoral programme in mathematics and science education at Université de Grenoble. She is member of the editorial and scientific committees of several research journals on mathematics teaching: *Educational Studies in Mathematics*, *Zentralblatt für Didaktik der Mathematik*, *For the Learning of Mathematics* and *Recherches en Didactique des Mathematiques*. She is member of the International Programme Committee for ICME-8, 1996.

During the academic year 1994-95 she is working as a fulltime researcher of the CNRS (National Center for Scientific Research) on "Communication and acquisition of scientific knowledge" in a research laboratory in Lyon.

Gilah C. Leder (Australia) is a professor in the Graduate School of Education at La Trobe University in Melbourne, Australia. Earlier appointments have included teaching at the secondary level, at the Secondary Teachers College (now Melbourne University), and at Monash University.

Her teaching and research interests embrace gender issues, factors which affect mathematics learning, exceptionality, and assessment in mathematics. She has published widely in each of these areas, including books on *Mathematics and gender* (1993, ed. with E. Fennema), *Assessment and learning of mathematics* (1992, ed.), *Quantitative methods in education research. A case study* (1992, with R. Gunstone), and *Educating girls. Pratice and research* (1989, ed. with E. Sampson).

Gilah Leder serves on various editorial boards and educational and scientific committees. She is president of the Mathematics Education Research Group of Australasia (MERGA). She is a frequent presenter at scientific and professional teaching meetings.

She is currently working on the following large research projects:
- o Learning mathematics in a social context
- o Exploration of the large Australian Mathematics Competition data base
- o Dimensions of gender/cultural conflict in mathematics teaching.

Carlos Vasco (Colombia) has a wide spectrum of mathematical interests. They include abstract algebra and category theory; exploration of different appproaches to logic; relations between culture, language, and mathematics; the study of the history and the philosophy of mathematics, and their implications in mathematics teaching and learning.

He was born in 1937 in Medellin, Colombia. He received his Licentiate in Philosophy and Letters at the Univdersidad Javerianna in Bogotá, Colombia. Later he obtained a Master's degree in theoretical physics and a Ph.D. (1968) in mathematics at Saint Louis University (St. Louis, Missouri, U.S.A.). He also studied theology in Frankfurt am Main, Germany, and became ordained priest as a member of the Society of Jesus (the Jesuit Order) in 1971.

Carlos Vasco is now full professor of mathematics at the National University of Colombia at Bogotá. In earlier years he has held several positions in USA and Colombia. Since 1978 he has been an advisor to the Ministry of Education of Colombia with the task of designing the elementary mathematics curriculum for grades 1 through 9. His main contribution has been the development of a comprehensive theoretical framework for the official curriculum reform in school mathematics, based on general systems theory and cognitive science. The main document was published in English in 1986 and in Spanish 1984, 1990, and 1991. During the years 1993 and 1994 he co-ordinated the ten-member Commission on Science, Education and Development appointed by the President of Colombia to design future government policy in these areas.

Finally, Carlos Vasco is a member of the Colombian National Academy of Sciences, the Colombian Society of Mathematics, and the Colombian Society for the Advancement of Science. He is a member of the editorial boards of *Educational Studies of Mathematics* and of the Colombian journal *Notas de Matemática*. He is a member of the International Programme Committee for ICME-8, 1996.

Dianzhou Zhang (China) was born in 1933. He completed his graduate work at East China Normal University (Shanghai) in 1956, where he went on to work as a lecturer and where he is now a professor of mathematics. In 1990 and 1991, he went to the United States. As a visiting professor he stayed at the City University of New York, State University of New York (Stony Brook), and the Mathematical Sciences Research Institute (Berkeley). During his stay he was a speaker at the Third UCSMP International Conference on Mathematics Education, Chicago University in 1991.

His research intèrests include functional analysis, history of mathematics and mathematics education. Before 1985 he published a number of papers on operator theory, and some books on the modern history of mathematics in the 20th century. However, when he took part in the ICME-6 in 1988 in Budapest, the importance of mathematics education left an immense impression on him. He then transfered a main part of his activity to the area of mathematics education.

In the 1990's, Dianzhou Zhang published ten books on mathematics education in China and organised a number of meetings in this field. In particular, the ICMI-China Regional Conference on mathematics education, held in Shanghai, August 1994, was organised under his leadership.

Dianzhou Zhang is a leading member of the education committee of the Chinese Mathematics Society. Moreover, he is the Editor-in-Chief of the Chinese Journal "Mathematical Teaching" which is one of the two most important magazines for mathematics education in China. He is also a member of the Executive Comittee of the Mathematical History Association of China. It is a major preoccupation of Dianzhou Zhang's to fight for a shift from "mathematics for some" to "mathematics for all" in China, and to serve as an advisor to many divisions of the National Commission for Education. Finally he is a member of the International Programme Committee for ICME-8, 1996.

Mogens Niss

ICMI Study:
Prespectives on the Teaching of Geometry for the 21st Century

Discussion Document

1. Why a study on geometry?

Geometry, considered as a tool for understanding, describing and interacting with the space in which we live, is perhaps the most intuitive, concrete and reality-linked part of mathematics. On the other hand geometry, as a discipline, rests on an extensive formalization process, which has been carried out for over 2000 years at increasing levels of rigour, abstraction and generality.

In recent years, research in geometry has been greatly stimulated by new ideas both from inside mathematics and from other disciplines, including computer science. At present, the enormous possibilities of computer graphics influence many aspects of our lives; in order to use these possibilities, a suitable visual education is needed.

Among mathematicians and mathematics educators there is a wide-spread agreement that, due to the manifold aspects of geometry, teaching of geometry should start at an early age, and continue in appropriate forms throughout the whole mathematics curriculum. However, as soon as one tries to enter into details, opinions diverge on how to accomplish the task. There have been in the past (and there persist even now) strong disagreements about the aims, contents and methods for the teaching of geometry at various levels, from primary school to university.

Perhaps one of the main reasons for this situation is that geometry has so many aspects, and as a consequence there has not yet been found - and perhaps there does not exist at all - a simple, clean, linear, "hierarchical" path from the first beginnings to the more advanced achievements of geometry. Unlike what happens in arithmetic and algebra, even basic concepts in geometry, such as the notions of angle and distance, have to be reconsidered at different stages from different viewpoints.

Another problematic point concerns the role of proofs in geometry: relations between intuition, inductive and deductive proofs, age of students at which proofs can be introduced, and different levels of rigour and abstraction.

Thus the teaching of geometry is not at all an easy task. But instead of trying to face and overcome the obstacles arising in the teaching of geometry, actual school-practice in many countries has simply bypassed these obstacles, cutting out the more demanding parts, often without any replacement. For instance, three-dimensional geometry has almost disappeared or has been confined to a marginal role in the curricula in most countries.

Starting from this analysis, and specifically considering the gap between the increasing importance of geometry for its own sake, as well as in research and in society, and the decline of its role in school curricula, ICMI feels that there is an urgent need for an international study, whose main aims are:

- To discuss the goals of the teaching of geometry at different school levels and according to different cultural traditions and environments.

- To identify important challenges and emerging trends for the future and to analyze their potential didactical impact.

- To exploit and implement new teaching methods.

2. Aspects of geometry

The outstanding historical importance of geometry in the past, in particular as a prototype of an axiomatic theory, is so universally acknowledged that it deserves no further comment. Moreover, in the last century and specifically during the last decades, as Jean Dieudonné asserted at ICME 4 (Berkeley, 1980), Geometry "bursting out of its traditional narrow confines [...] has revealed its hidden powers and its extraordinary versatility and adaptability, thus becoming one of the most universal and useful tools in all parts of mathematics" (J. Dieudonné: The Universal Domination of Geometry, ZDM 13 (1) , p 5-7 (1981)).

Actually, geometry includes so many different aspects, that it is hopeless (and maybe even useless) to write out a complete list of them. Here we mention only those aspects, which in our opinion are particularly relevant in view of their didactical implications:

- Geometry as *the science of space*. From its roots as a tool for describing and measuring figures, geometry has grown into a theory of ideas and methods by which we can construct and study idealized models of the physical world as well as of other real world phenomena. According to different points of view, we get euclidean, affine, descriptive, projective geometry, but also topology or non euclidean and combinatorial geometries.

- Geometry as *a method for visual representations of concepts and processes* from other areas in mathematics and in other sciences. E. g. graphs and graph theory, diagrams of various kinds, histograms.

- Geometry as *a meeting point* between mathematics as a theory and mathematics as a model resource.

- Geometry as *a way of thinking and understanding* and, at a higher level, as *a formal theory*.

- Geometry as *a paradigmatic example for teaching deductive reasoning.*

- Geometry as *a tool in applications*, both traditional and innovative. The latter ones include e. g. computer graphics, image processing and image manipulation, pattern recognition, robotics, operations research.

Another distinction should be made with respect to several different approaches according to which one may deal with geometry. Roughly speaking, possible approaches are:

* manipulative

* intuitive

* deductive

* analytic.

Also one may distinguish between a geometry which stresses "static" properties of geometric objects and a geometry where objects are considered in a "dynamic" setting, as they change under the effect of different types of space transformations.

3. Is there a crisis in the teaching of geometry?

During the second half of this century geometry seems to have progressively lost its former central position in mathematics teaching in most countries. The decrease has been both qualitative and quantitative. Symptoms of this decrease may be found for instance in recent national and international surveys on the mathematical knowledge of students. Often geometry is totally ignored or only a very few items concerned with geometry are included. In the latter case questions tend to be confined to some elementary "facts" about simple figures and their properties, and performance is reported to be relatively poor.

What are the main causes of this situation?

- From about 1960 to 1980 a general time pressure on traditional topics has occurred, due to the introduction of new topics in mathematics curricula (e.g. probability, statistics, computer science, discrete mathematics). At the same time the number of school hours devoted to mathematics has gone down. The "modern mathematics movement" has contributed - at least indirectly - to the decline of the role of euclidean geometry, favouring other aspects of mathematics and other points of view for its teaching (e.g. set theory, logic, abstract structures). The decline has involved in particular the role of visual aspects of geometry, both three-dimensional and two-dimensional, and all those parts which did not fit into a theory of linear spaces as, for instance, the study of conic sections and of other noteworthy curves.

- In more recent years there has been a shift back towards more traditional contents in mathematics, with a specific emphasis on problem posing and problem solving activities. However, attempts to restore classical euclidean geometry - which earlier in many parts of the world was the main subject in school geometry - have so far not been very successful. The point is that in traditional courses on euclidean geometry the material is usually presented to students as a ready-made end product of mathematical activity. Hence, in this form, it does not fit well into curricula where pupils are expected to take an active part in the development of their mathematical knowledge.

- In most countries the percentage of young people attending secondary school has increased very rapidly during the last decades. Thus the traditional way of teaching abstract geometry to a selected minority has become both more difficult and more inappropriate for the expectations of the majority of students of the new generations. At the same time, the need for more teachers has caused, on average, a decline in their university preparation, especially with respect to the more demanding parts of mathematics, in particular geometry. Since younger teachers have learned mathematics under curricula that neglected geometry, they lack a good background in this field, which in turn fosters in them the tendency to neglect the teaching of geometry to their pupils.

The situation is even more dramatic in those countries which lack a prior tradition in schooling. In some cases geometry is completely absent from their mathematics curricula.

- The gap between the conception of geometry as a research area and as a subject to be taught in schools seems to be increasing; but so far no consensus has been found how to bridge this gap, nor even whether it could (or should) be bridged through an introduction of more advanced topics in school curricula at lower grades.

4. Geometry as reflected in education

In former sections, we have considered geometry mainly as a mathematical theory and have analyzed some aspects of its *teaching*. Since *learning* is unquestionably the other essential pole of any educational project, it is now appropriate to pay due attention to the main variables which may affect a coherent teaching/learning process. Consequently, several different aspects or "dimensions" (considered in their broadest meaning) must be taken into account:

- *The social dimension*, with two poles:
    - The cultural pole, i.e. the construction of a common background (knowledge and language) for all the people sharing a common civilization;
    - The educational pole, i.e. the development of criteria, internal to each individual, for self consistency and responsibility.

- *The cognitive dimension*, i.e. the process which, starting from reality, leads gradually to a refined perception of space.

- *The epistemological dimension*, i.e. the ability to exploit the interplay between reality and theory through modelling (make previsions, evaluate their effects, reconsider choices). Thereby axiomatization enables one to get free from reality; this in turn may be seen as a side-step which allows further conceptualization.

- *The didactic dimension*, i.e. the relation between teaching and learning. Within this dimension several aspects deserve consideration. As an example, we list three of them:

    - To make various fields interact (both within mathematics and between mathematics and other sciences).

    - To make sure that the viewpoints of the teacher and the pupils are consistent in a given study. For instance, to be aware that different distance scales may involve different conceptions and processes adopted by the pupils, even though the mathematical situation is the same: in a "space of small objects", visual perception may help to make conjectures and to identify geometric properties; when dealing with the space where we are used to move around (the classroom, for instance) it is still easy to get local information, but it may be difficult to achieve an overall view; in a "large scale space" (as is the case in geography or in astronomy) symbolic representations are needed in order to analyze its properties.

    - To pay due consideration to the influence of tools available in teaching/learning situations (from straightedge and compass, as well as other concrete materials, to graphic calculators, computers and specific software).

It goes without saying that all these dimensions are interrelated with each other and that they should also be related appropriately to different age levels and school types: pre-primary level, primary level, lower secondary level, upper secondary level (where differentiation into academic, technical, vocational tracks usually starts), tertiary (i.e. university) level, including teacher preparation.

5. New technology and teaching aids for geometry

There is a long tradition of mathematicians making use of technological tools, and conversely the use of these tools has given rise to many challenging mathematical problems (e.g. straightedge and compass for geometric constructions, logarithms and mechanical instruments for numerical computations). In recent years new technology, and in particular computers, has affected dramatically all aspects of our society. Many traditional activities have become obsolete, while new professions and new challenges arise. For instance, technical drawing is no longer done by hand. Nowadays, instead, one uses commercial software, plotters and other technological devices. CAD/CAM and symbolic algebra software are becoming widely available.

Computers have also made it possible to construct "virtual realities" and to generate interactively animations or marvellous pictures (e.g. fractal images). Moreover, electronic devices can be used to achieve experiences that in everyday life are either inaccessible, or accessible only as a result of time-consuming and often tedious work.

Of course, in all these activities geometry is deeply involved, both in order to enhance the ability to use technological tools appropriately, and in order to interpret and understand the meaning of the images produced.

Computers can be used also to gain a deeper understanding of geometric structures thanks to software specifically designed for didactical purposes. Examples include the possibility of simulating traditional straightedge and compass constructions, or the possibility of moving basic elements of a configuration on the screen while keeping existing geometric relationships fixed, which may lead to a dynamic presentation of geometric objects and may favour the identification of their invariants.

Until now, school practice has been only marginally influenced by these innovations. But in the near future it is likely that at least some of these new topics will find their way into curricula. This will imply great challenges:

- How will the use of computers affect the teaching of geometry, its aims, its contents and its methods?

- Will the cultural values of classical geometry thereby be preserved, or will they evolve, and how?

- In countries where financial constraints will not allow a massive introduction of computers into schools in the near future, will it nevertheless be possible to restructure geometry curricula in order to cope with the main challenges originated by these technological devices?

6. Key issues and challenges for the future

In this section we list explicitly some of the most relevant questions which arise from the considerations outlined in the preceding sections. We believe that a clarification of these issues would contribute to a significant improvement in the teaching of geometry. Of course we do not claim that all the problems quoted below are solvable, nor that the solutions are unique and have universal validity. On the contrary, the solutions may vary according to different school levels, different school types and different cultural environments.

1. Aims

Why is it advisable and/or necessary to teach geometry?

Which of the following may be considered to be the most relevant aims of the teaching of geometry?

- To describe, understand and interpret the real world and its phenomena

- To supply an example of an axiomatic theory

- To provide a rich and varied collection of problems and exercises for individual student activity

- To train learners to make guesses, state conjectures, provide proofs, and find out examples and counterexamples

- To serve as a tool for other areas of mathematics

- To enrich the public perception of mathematics.

2. Contents

What should be taught?

Is it preferable to emphasize "breadth" or "depth" in the teaching of geometry? And is it possible/advisable to identify a core curriculum?

In the case of an affirmative answer to the second question above, which topics should be included in syllabi at various school levels?

In the case of a negative answer, why is it believed that teachers or local authorities should be left free to choose the geometric contents according to their personal tastes (is this point of view common to other mathematical subjects, or is it peculiar to geometry)?

Should geometry be taught as a specific, separate subject, or should it be merged into general mathematical courses?

There seems to be widespread agreement that the teaching of geometry must reflect the actual and potential needs of society. In particular, geometry of three-dimensional space should be stressed at all school levels, as well as the relationships between three-dimensional and two-dimensional geometry. How could and should the present situation (where only two-dimensional geometry is favoured) therefore be modified and improved?

In which ways can the study of linear algebra enhance the understanding of geometry? At what stage should "abstract" vector space structures be introduced? And what are the goals?

Would it be possible and advisable also to include some elements of non euclidean geometries into curricula?

3. Methods

How should we teach geometry?

Any topic taught in geometry can be located somewhere between the two extremes of an "intuitive" approach and a "formalized" or "axiomatic" approach. Should only one of these two approaches be stressed at each school level, or should there be a dialectic interplay between them, or else should there be a gradual shift from the former to the latter one, as the age of students and the school level progresses?

What is the role of axiomatics within the teaching of geometry? Should a complete set of axioms be stated from the beginning (and, if so, at what age and school level) or is it advisable to introduce axiomatics gradually, e. g. via a "local deductions" method?

Traditionally, geometry is the subject where "one proves theorems". Should "theorem proving" be confined to geometry?

Would we like to expose students to different levels of rigour in proofs (as age and school level progress)? Should proofs be tools for personal understanding, for convincing others, or for explaining, enlightening, verifying?

Starting from a certain school level, should every statement be proved, or should only a few theorems be selected for proof? In the latter case, should one choose these theorems because of their importance within a specific theoretical framework, or because of the degree of difficulty of their proof? And should intuitive or counterintuitive statements be privileged?

It seems that there is an international trend towards the teaching of analytic methods in increasingly earlier grades, at the expense of other (synthetic) aspects of geometry. Analytic geometry is supposed to present algebraic models for geometric situations. But, as soon as students are introduced to these new methods, they are suddenly projected into a new world of symbols and calculations in which the link between geometric situations and their algebraic models breaks down and geometric interpretations of numerical calculations are often neglected. Hence, at what age and school level should teaching of analytic geometry start? Which activities, methods and theoretical frameworks can be used in order to restore the link between the algebraic representation of space and the geometric situation it symbolizes?

How can we best improve the ability of pupils to choose the appropriate tools for solving specific geometric problems (conceptual, manipulative, technological)?

4. Books, computers, and other teaching aids

Are traditional textbooks as appropriate for teaching and learning geometry as we would like them to be?

How do teachers and pupils actually use geometry textbooks and other teaching aids? How would we like pupils to use them?

What changes could and should be made in teaching and learning geometry in the perspective of increased access to software, videos, concrete materials and other technological devices?

What are the advantages, from the educational and geometrical point of view, that can follow from the use of such tools?

Which problems and limitations may arise from the use of such tools, and how can they be overcome?

To what extent is knowledge acquired in a computer environment transferable to other environments?

5. Assessment

The ways of assessment and evaluation of pupils strongly influence teaching and learning strategies. How should we set out objectives and aims, and how should we construct assessment techniques that are consistent with these objectives and aims? Are there issues of assessment which are peculiar to the teaching and learning of geometry?

How does the use of calculators, computers and specific geometric software influence examinations as regards content, organization and criteria for the evaluation of the answers of the students?

Should assessment procedures be based mainly upon written examination papers (as it seems to be customary in many countries) or else what should be the role of oral communication, of technical drawing and of work with the computer?

What is it exactly that should be evaluated and considered for assessment: The solution outcome? The solution process? The method of thinking? Geometric constructions?

6. Teacher preparation

One essential component of an efficient teaching/learning process, is good teacher preparation, as regards both disciplinary competence and educational, epistemological, technological and social aspects. Hence, what specific preparation in geometry is needed (and realistically achievable) for prospective and practicing teachers?

It is well known that teachers tend to reproduce in their profession the same models they have experienced when they were students, regardless of subsequent exposure to different points of view. How is it then possible to motivate the need for changes in the perspective of teaching geometry (both from the content and from the methodological point of view)?

Which teaching supplies (books, videos, software,...) should be made available for in-service training of teachers, in order to favour a flexible and open-minded approach to the teaching of geometry?

7. Evaluation of long-term effects

All too often the success (or failure) of a curricular and/or methodological reform or innovation for a certain school system is evaluated on the basis only of a short period of observation of its outcomes. Moreover usually there are no comparative studies on the possible side effects of a change of content or methods. Conversely, it would be necessary to look also at what happens in the long term. For instance:

- Does a visual education from a very young age have an impact on geometric thinking at a later stage?

- How does an early introduction of analytic methods in the teaching of geometry influence the visual intuition of pupils? When these pupils become professionals, do they rely more on the intuitive or on the rational parts of the geometry teaching to which they have been exposed?

- What is the impact of an extensive use of technological tools on geometry learning?

8. Implementation

At ICME 5 (Adelaide, 1984) J. Kilpatrick asked a provocative question: What do we know about mathematics education in 1984 that we did not know in 1980? Recently the same question has been picked up again in the ongoing ICMI study: "What is research in mathematics education, and what are its results". As for geometry, the possibility of relying on research results would be extremely useful in order to avoid reproposing in the future paths already proved unsuccessful, and conversely in order to benefit from successful solutions. And, as for still unsettled and relevant questions, we would like research to give us useful information in order to clarify the advantages and drawbacks of possible alternatives.

Hence, a key question might be:

What do we already know from research about the teaching and learning of geometry and *what would we want future research to tell us?*

7. Call for papers

The ICMI study **"Perspectives on the Teaching of Geometry for the 21st Century"** will consist of an invited *Study Conference* and a *Publication* to appear in the ICMI study series, based on the contributions to, and the outcomes of, the Conference.

The Conference is scheduled for 28 September - 2 October 1995 in Catania (Italy). The International Program Committee (IPC) for the study hereby invites individuals and groups to submit ideas, suggestions and contributions on major problems or issues related to this discussion document, no later than February 15, 1995.

Although participation in the conference requires an invitation from the IPC, "experts" and "newcomers" interested in contributing to and participating in the conference are encouraged to contact the chair of the IPC. Unfortunately, an invitation to attend does not imply that financial support will be provided by the organizers.

Papers, as well as suggestions concerning the content of the study conference program should be sent to

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Mathematics for Work

Susan L. Forman and Lynn Arthur Steen

I hear and I forget;
I see and I remember;
I do and I understand.

Mathematics in the schools serves multiple purposes, including preparation for work, for citizenship, for general education, and for higher education. Of these, preparation for productive work takes priority in the minds of parents and the public. Indeed, in this era of advanced technology, global competitiveness, and electronic telecommunications, more students than ever before need to learn more mathematics than anyone has every tried to teach on so massive a scale. Despite differences in educational traditions, all nations now face a similar challenge — of preparing students for a rapidly changing world economy.

In the United States, this conviction is creating strong pressure for more pragmatic work-oriented programs in the schools. Indeed, all students will eventually be employed. This pressure offers mathematics education a rich opportunity to rethink the traditional school curriculum in order to better integrate academic and work-related mathematics. But it also runs a serious risk of deleterious side effects, notably the rebirth of second-class tracking under the guise of preparing students for twenty-first century jobs.

To begin to address these issues, the Mathematical Sciences Education Board (MSEB) recently convened at the U.S. National Academy of Sciences a meeting of vocational and mathematical educators with mathematicians, industrial leaders, and policy makers to discuss key issues such as articulation, tracking, student needs, and core curriculum. The issues that are unfolding through these discussions in the United States provide a useful case study that contains lessons from which others can learn — and which might serve as a prelude to some future ICMI Study.

Preparing for an International Economy

In a recent study entitled "Preparing for the Workplace," the National Research Council reports that only one in five U.S. workers holds a (four-year) baccalaureate degree. The other 80% enter the world of work either directly out of high school, after beginning but not completing a four-year college program, or after completing some other form of post-secondary education in institutions ranging from community colleges to employer-sponsored institutes. Ironically, increasing numbers of university degree holders cannot find employment based on their degree, even as industry claims to be unable to hire workers with the technical skills needed for many jobs.

The jobs available to this "other 80%" depend increasingly on mathematical thinking. Many corporations world-wide are moving to a flat management structure in which workers are responsible for their own quality control and for their own management through small employee-led teams. Workers no longer have the luxury of just doing what they are told. They are now expected to develop their own problem-solving strategies, often using statistical or mathematical tools suitable to a data-rich high technology work environment. What used to be called "blue collar" jobs are quietly but inexorably being transformed by technology into "white smock" jobs.

In response to this challenge, U.S. officials now strongly advocate greater emphasis on educational programs that prepare students for technical careers in fields ranging from telecommunications to manufacturing, from nursing to agriculture. These programs fall under the general titles of "tech-prep," "school-to-work," "education for employment," or "advanced technological education." Many combine the last two years of high school with the first two years of post-secondary education into "2+2" programs; some expand to form "2+2+2" programs.

In addressing a similar set of issues, the Topic Group on "Mathematics for Work" at ICME-7 distinguished two levels of education for work: that for "qualified workers" and that for professionals. The current U.S. discussion about preparing an internationally competitive technical workforce is more about the former than the latter, but the major educational challenges lie in building effective transitions from one to the other.

Ensuring Options

Educational traditions in the United States differ in many respects from those in other nations, not least in how we handle transitions from school to work. In nations with well-established apprenticeship systems, the route from study to employment is transparent to students and their parents: everyone can foresee the consequences of choices made at various stages in the educational system. In the United States, however, linkages between school and work are virtually absent, and the various options are opaque not only to students and parents, but also to educators and employers.

In addition, the multi-cultural character of the United States imposes perhaps more heavily than in other nations a demand that education be structured so as to keep student options open for as long as possible. It is an axiom of U.S. educational policy, repeatedly asserted by U.S. Labor Secretary Robert Reich, that all educational programs should prepare students for post-secondary study.

Yet by increasing the emphasis on preparation for work, schools may create a new and apparently more "respectable" form of tracking — a deeply entrenched tradition of U.S. schools that relegated large numbers of students, disproportionately minorities, to dead-end courses in which they received little challenge and even less education. New career-based tracking will be more seductive — who can resist the allure of high-tech occupational preparation? — but equally capable of perpetuating socio-economic class distinctions. Decisions made in early high school years — especially decisions about mathematics courses — can program students into a tech-prep or college-prep curriculum with little opportunity for switching. Yet students' disposition for change between ages 14 and 20 virtually guarantees that no inflexible system of early tracking can be personally or educationally sound.

Setting Standards

Another way in which the United States differs from many other nations is in how it goes about setting educational standards. In most nations, a central governmental authority sets broad standards for education in mathematics and other subjects. In the U.S., however, it was not the government but mathematics educators acting through the National Council of Teachers of Mathematics (NCTM) who developed national Standards for school mathematics. These Standards

(a) call for a broader curriculum that encompasses the full reach of mathematics in practice;
(b) stress active, engaged learning of mathematics situated in authentic contexts;
(c) argue against dead-end tracking that disenfranchises some children by denying them access to important mathematics; and
(d) urge widespread use of technology.

These NCTM priorities are remarkably similar to those identified in the ICME-7 Topic Group on "Mathematics for Work": the need to adapt mathematics teaching to a technological environment; the need to focus instruction on "situated knowledge" — on mathematics embedded in the context of work; and the need to avoid abuses such as dead-end tracking and inappropriate use of mathematics as a filter for jobs. At the level of rhetoric, these international expectations appear entirely consistent with those articulated for the United States by the NCTM.

However, educators are not the only (and perhaps not even the primary) developers of standards. Industrial associations in the U.S. are now creating occupational standards in "skills clusters" to provide portable national credentials that will fit the workplace better than the traditional high school or college diploma. These industry-driven expectations have a different character than those developed by educators: they stress general work-place expectations such as punctuality, reliability and teamwork, and technical expectations such as precision and zero-error performance. In many ways, true rigor emerges when students seriously prepare themselves to meet industry standards.

Preparing for Work

Comparison of mathematics in the workplace with mathematics in the classroom reveals a disjuncture that is disconcerting to anyone who believes that a primary purpose of school is to prepare students for work. School mathematics lives in a decontextualized ether, employing data that are without blemish and language that is devoid of ambiguity. In contrast, real problems are embedded in concrete tasks, use data that are often ill-defined or inaccurate, and rely on language that is often imprecise and misleading. In the world of work, mathematics is collaborative rather than individualistic; accuracy is defined by the situation rather than given by the text-book; and mathematical processes are used rather than studied. The new challenge is to seek common ground among these very different traditions — of mathematics for and from the workplace and of mathematics as preparation for further study.

One resolution of the dilemma of tracking would be a common mathematics program that could serve equally well as preparation both for college and for skilled work. All students could benefit from the broadening effects of such a high school preparation, yet there are currently few good models of curricula that serve both agendas. Another approach would be to develop a new form of vocational and technical education, with status equal to the academic track, that would simultaneously prepare students for the world of work and for further study in post-secondary institutions. U.S. educators who are concerned about vocational education debate both the desirability and feasibility of such a "separate but equal" track.

Achieving Equal Status

Efforts to bridge the vocational-academic gap often founder on matters of status. Because of their low status in the U.S., vocational programs often attract the least able students and teachers. Public support for vocational programs is rooted in the conviction that they are always for someone else's children. Neither teachers nor parents nor students believe vocational programs are or can be of comparable value to the regular "academic" curriculum. Indeed, as currently constituted, vocational courses frequently provide students with only a thin veneer of mathematical understanding. All too often, such programs operate within a narrow band of educational goals, providing little of value that transfers to a variety of jobs. The working knowledge that students need involves much more than technical skills; a well-designed educational program requires a learning trajectory in which students move from simple to more complex tasks, and from activities that are specialized to those that are central to the work enterprise. One important trend that addresses these needs is the shift from preparation for particular trades (e.g., carpentry) to education about all aspects of an industry (e.g., construction), including economics, history, science, and, of course, mathematics.

In order to avoid the stigma imposed by the academic caste system, many in the U.S. now argue that vocational education should be seen not as a path to a different goal, but as a different path to the same (academic) goal. According to this view, the central purpose of vocational education is not to train students for a particular job, but to offer students an alternative path to prepare for advanced academic training. This interpretation, which emphasizes vocational education for all students, not just at-risk students, might avoid many of the weaknesses often associated with occupational programs. These changes are important since, if it is to be accepted as central to schooling, vocational education needs to remain close to school reform.

Cognitive Apprenticeships

An interesting contrast emerges in comparing the NCTM Standards with educational practice in strong vocational programs. The Standards call for both broad mathematics and active pedagogy. Among mathematics teachers one can more readily find examples of the former than of the latter, whereas among vocational teachers the reverse is true: vocational pedagogy almost always exemplifies the ideals of the Standards, while content too often lags behind. Historically, apprenticeships such as carpentry and tailoring involved physical activities that could be easily observed and learned by imitation and practice. Today, many trades (e.g., medical technicians) operate at symbolic and cognitive levels that depend on reasoning strategies ordinarily hidden from view. The teacher's job in this new environment is to develop classroom practices that put students into "cognitive apprenticeships" in which they come to understand rather than merely imitate the work of experts. To be effective as preparation for life-long learning, vocational programs must include abstract principles, varied situations, and general procedures that allow learners to adapt to new situations.

Responding to the Challenge

Educational institutions, being inherently conservative, have great difficulty responding to the multiple challenges posed by a rapidly changing workplace. Fortunately, in the United States, this challenge has been taken up with imagination and flexibility by two-year or community colleges.

These colleges operate within commuting distance of 95% of the population of the United States and often serve a population of older, minority, and low socio-economic students. Typically, they offer two types of programs: "associate" degrees that provide either a mid-point milepost for students who intend to continue on for a four year degree or an applied credential for students preparing for jobs in technical areas; and "certificates" that respond to employers' needs for specialized programs.

Two-year colleges have grown in enrollment by over 250% during the past two decades, and offer some of the most exciting new routes into the emerging economy. These schools are the prominent shapers of tomorrow's front-line workers in occupations such as dental hygiene, marketing, engineering technology, and agribusiness. Two-year colleges currently serve as the hub of a growing system of vocation-oriented education that links high schools, industry, and universities. Indeed, the National Science Foundation has targeted these colleges for major new funding since they offer an effective crucible for the necessary curricular development. Mathematics is a central yet underdeveloped component of these rapidly growing programs.

Next Steps

Mathematicians who think about curricular issues typically focus on the "academic track" that leads to scientific, engineering, and mathematics courses at the university level. Failure by university mathematicians and mathematics educators to recognize and employ the significant mathematics embedded in workplace applications has led to divergence between the kind of mathematics taught in schools and that which is useful in the workplace.

Similarly, those who develop curricula for technical and vocational programs rarely work with mathematicians or mathematics educators to ensure consistency with the expectations and standards of mathematics education. This has created an urgent need to improve communication among the leaders of the different communities, since most of the school-to-work curriculum leaders have little or no background in mathematics education and few mathematics educators have any experience in work outside of education.

The increasing political pressure for more utilitarian alternatives to the traditional pre-university curriculum provide mathematicians with a marvelous opportunity to demonstrate the universal applicability of their discipline — to show that it is good for something other than just preparing more university professors. An examination of these issues may be useful for ICMI — perhaps as a prelude to some future ICMI Study on Technical Education. The issues are vast, encompassing aspects of mathematical literacy, adult education, and the very nature of school mathematics for all students.

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The Methodology of Mathematics

Ronald Brown and Timothy Porter

Introduction

This essay is based on a talk given by the first author to students and staff of the Departmento de Geometria e Topologia at the University of Seville in November, 1993. The issues presented there have been part of a continued debate and discussion at Bangor over many years, and this explains why this is a joint paper.

The aim of the talk, and the reason for discussing these topics, was to give students an understanding and a sense of pride in the aims and achievements of their subject, and so help them explain these aims and achievements to their friends and relatives. This pride in itself would be expected to contribute to their enjoyment of the subject, whatever their own level of achievement. Because of this, and because of its origin, the tone of the article is principally that of an address to students.

We start with some general questions to which we believe it is helpful for students to be able to formulate some kind of answers. The question for teachers of mathematics at all levels is to what extent, if at all, the training of mathematicians should involve professional discussion of, and assessment in, possible answers to these questions, such as those given of suggested here.

Some basic issues for mathematicians

1) Is mathematics important? If so, for what, in what contexts, and why?
2) What is the nature of mathematics, in comparison with other subjects?
3) What are the objects of study of mathematics?
4) What is the methodology of mathematics, what is the way it goes about its job?
5) Is there research going on in mathematics? If so, how much? What are its broad aims or main aims? What are its most important achievements? How does one go about doing mathematical research?
6) What is good mathematics?

It may be thought by some that these questions are beside the point, a waste of time, and not what real mathematicians should be considering. Against this we would like to give a quotation from Albert Einstein (1916), translated in the *Mathematical Intelligencer* 12 (1990) no. 2, p. 3:

> "How does a normally talented research scientist come to concern himself with the theory of knowledge? Is there not more valuable work to be done in his field? I hear this from many of my professional colleagues; or rather, I sense in the case of many more of them that this what they feel.

I cannot share this opinion. When I think of the ablest students whom I have encountered in teaching - i.e. those who have distinguished themselves by their independence and judgement and not only mere agility - I find that they have a lively concern for the theory of knowledge. They like to start discussions concerning the aims and methods of the sciences, and showed unequivocally by the obstinacy with which they defend their views that this subject seemed important to them.

This is not really astonishing. For when I turn to science not for some superficial reason such as money-making or ambition, and also not (or at least not exclusively) for the pleasure of the sport, the delight og brain-athletics, then the following questions must burningly interest me as a disciple of science: What goal will be reached by the science to which I am dedicating myself? To what extent are its general results "true"? What is essential and what is based only on the accidents of development? ...

Concepts which have proved useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts. They then become labelled as "conceptual necessities", "a priori situations", etc. The road of scientific progress is frequently blocked for long periods by such errors. It is therefore not just an idle game to exercise our ability to analyze familiar concepts, and to demonstrate the conditions on which their justification and usefulness depend, and the way in which these developed, little by little ..."

There are a number of reasons apart from authority to consider the above questions. A professor of mathematics in the UK with whom we discussed these questions suggested that the aim of considering them was to get students to reflect on the methods of mathematics. He remarked, as if seeing this for the first time, that there was a well known difference between human beings and other animals, that humans have this ability to reflect on what they do, and that this ability affects beneficially a lot of human activity. One aspect of this reflection is that it leads to the notion of value judgement, again a facility which humans have which is not apparently shared by other animals, or at least not in a way in which we can communicate, by and large.

Reflection on an activity is, generally, a useful way of increasing its effectiveness, as we are able to analyze what is essential, what is important, and how the activity can be done avoiding the easiest of mistakes in method. On these grounds it is reasonable that we should reflect on the activity of mathematics. In reflection, we also usually are aware of the value of the activity.

Another reason for our considering these questions was through a comparison with aspects of education in art. We have heard it argued that art education is considerably ahead of science education in arousing the interest and independence of students, so it is worth considering how art educators go about things. Here then are aims which have been given for a course in design:

1) To teach students the principles of good design
2) To encourage independence and creativity
3) To give students a range of practical skills so that they can apply the principles of good design employment

Is there something here from which mathematics course can learn? Is it reasonable aims for a mathematics course to replace in the above the word "design" by the word "mathematics"? If not, why not?

Here is another quotation form the book T. Dantzig, *Number: the language of science*:

> "This a book on mathematics: it deals with symbols and form and with ideas which are back of the symbol or the form.
>
> The author holds that our current school curricula, by stripping mathematics of its cultural content and leaving a bare skeleton of technicalities, have repelled many a fine mind. It is the aim of this book to restore this cultural content and present the evolution of mathematics as the profoundly human story it is."

Is there something in this from the point of view of a higher level of teaching of mathematics? This book dates from the 1930s. Have we made much progress since then in dealing with the point he raises?

Now let us consider the questions one by one.

What is the importance of mathematics?

It is not generally recognized how much of a part mathematics plays in our daily lives. Some of the mathematics is of course quite old: every day we use numbers, graphs, addition and multiplication. It is easy to forget that the invention of these was at one time a great discovery. The replacement of Roman numerals by Arabic numerals, and so the possibility of a good book-keeping system, is said to have led to the prosperity of Venice in the 14th century. It is also of interest here to note the importance of pedantry in mathematics. A key aspect of the Arabic system is its use of the number zero. At first it seems absurd to count the number of objects in an empty box. The surprise is how essential this is for an adequate numeration system, in which the number 0 is used as a place marker. The lack of this concept of zero held up the progress of mathematics for centuries.

On a higher level, without the mathematics of error correcting codes we would not have had the beautiful pictures of Jupiter from the Voyager II. This mathematics is also essential in many aspects of telecommunications and of computers, and in particular for CD players. There is an amusing story about this last application. Negotiations between Sony and the Dutch company Philips about the standards for CD were held at top management. The Japanese considered Philips' proposal for error correction inferior to theirs, and in the end the Japanese proposal was accepted. Back in Eindhoven, the embarrassed managers called in their science directors to declare that the company did not have sufficient expertise in this area called "coding theory" and to find out where in Europe the real experts could be found. The their dismay, the answer was: "in Eindhoven!", in the person of the Dutch number theorist Jack van Lint!

Without the mathematics of cryptography, there would not be possible the current level of electronic financial transactions crossing the world, and involving billions of dollars. Currently, the mathematics of category theory, a theory of mathematical structures, is being used to give new insights into future logics and algebras for the design of the next generation of programs and software.

The enormous applications of mathematics in engineering, in statistics, in physics, are common knowledge. It is also imagined that the role of mathematics is being taken over by the use of supercomputers. It is not so generally realized that these supercomputers are the servants of mathematical and conceptual formulations: the electronics is marvelous in that it does the calculations so quickly and accurately. For example, body scanners are an application, a realization, of a piece of the 19th century mathematics expressing how to reconstruct a solid object of varying density from views through it of on X-ray, where the only measurement is the change of intensity as the ray passes through the body, for a large number of varying positions of the ray. The theories of the big bang, of fundamental particles, would not be possible without mathematics.

What is the nature of mathematics?

There is here a mystery. The Noble Prize-winner E. Wigner has written a famous essay "The unreasonable effectiveness of mathematics in the physical sciences". For us, the key word is "unreasonable". He is talking about the surprise that the use of mathematics is able to give predictions which are in accord with experiment to the extent of nine significant figures. How is such astonishing accuracy possible?

It seems likely that a full "explanation" of the success of mathematics would need more understanding of language, of psychology, of the structure of the brain and its action, than is at present conceivable. Even worse the development of such understanding might need, indeed must need, a new kind and type of mathematics. It is still important to analyze the scope and limitations of mathematics. It is also reasonable that such an analysis should be a necessary part of the education and assessment of a student of mathematics. Of what use is a student who does not know such things?

Here then are some quotations from this article:

> "... that the enormous usefulness of mathematics in the physical sciences is something bordering on the mysterious, and that there is no rational explanation for it."
>
> "Mathematics is the science of skilful operations with concepts and rules invented just for this purpose." [this purpose being the skilful operation ...]
>
> "The principal emphasis is on the invention of concepts."
>
> "The depth of thought which goes into the formation of mathematical concepts is later justified by the skill with which these concepts are used."
>
> "The statement that the laws of nature are written in the language of mathematics was properly made three hundred years ago; [it is attributed to Gallileo] it is now more true than ever before."

> "The observation which comes closest to an explanation for the mathematical concepts' cropping up in physics which I know is Einstein's statement that the only physical theories which we are willing to accept are the beautiful ones. It stands to argue that the concepts of mathematics, which invite the exercise of so much wit, have the quality of beauty."

In order to discuss this, it is of interest to compare mathematics with other subjects, and to link this with the question of the objects of study, and of importance.

Suppose we ask questions of a few of our fellow scientists as to why one should study their subject. Answers might run as follows:

The astronomer: In astronomy we study the beginnings of the universe, and the flow of time over billions of years, as well as the furthest distances of space. What could be more enthralling? We have some money for this study, with various telescopes over the world, but of course not enough.

The physicist: In physics, we study the fundamental constituents of matter. What could be more fascinating? Without physics, there would be no astronomy, for example. Thus many more of the best students should study physics, and the Government should give us a lot of money.

The chemist: In chemistry, we make molecules do things for us, so that chemistry is part of everyday lives. Without the understanding found by chemistry, there would be no study of the stars, and no understanding of biology, of the formation of planets. So, many of the brightest students should study chemistry, and the Government should give us a lot more money.

The biologist: Biology is about life. What is life? How did it come about? How does it interact with us and the world? We are all concerned with life. So, many more students should study biology, and the Government should give us a lot more money.

The engineer: Engineering is about making things which control our environment and do things for us. Without engineering, modern civilization is inconceivable. Many more students should study engineering, and the Government should give us a lot more money.

Of course there a many more protagonists in this story. Also, we have exaggerated the concern with finance. Yet, the financing of an activity is one measure of the importance attached to it.

Let us turn now to the mathematician, and ask for his or her story and justification for existence in the hustle and struggle for a place in the scheme of things. It is quite possible that from even a well known mathematician you will get no clear answer. It might be claimed as an important achievement that, for example, Fermat's last theorem is now solved, or at least solved apart from some residual worries, some 200 years after the problem was posed. Will such a solution, however, bring in the crowds or the cash? Why should it? Certainly, this solution was an achievement, but what is its general import? Why was so much effort devoted to it? Is it merely comparable with breaking another record?

These questions are not idle. Resources are limited. Any one person's interests are limited. We need a more convincing answer for the support of our subject, and to persuade people to study it. Here is our try:

The mathematician: Mathematics is about the study of *pattern and structure*, and the logical analysis and calculation with patterns and structures. In our search for understanding of the world, driven by the need for survival, and simply for the wish to know what is there, and to make sense of it, we need a science of structure, in the abstract, and a method of knowing what is true, and what is interesting, for these structures. Thus mathematics in the end underlies and is necessary for all these other subjects. This is part of our claim for your attention, and for the support of our studies. Another part of this claim is the fascination and wonder at the new patterns and structures the surprising relationships, which our study has found. Mathematics is also bringing humility. We know how hard it can be to decide the truth of but on apparently simple and clear statement. We are aware of the limitations of mathematical truth, that not all that is true can be proved, as shown by the undecidability results of Gödel. You will not find a mathematician writing that the final solution, the unified theory which will solve everything, is at hand. Rather, we are looking for the surprises which show us a new view of the world, and new riches to explore. Experience leads us to expect these to appear. For the mathematician, the world is not only stranger than you imagine, but stranger than you can now imagine. It is our job to investigate this strangeness.

What are the objects of study of mathematics?

This has already been answered to some extent. Mathematics does not study things, but the relations between things. A description of such a relation i what we mean by a "concept". Thus we talk about the distance between towns, and might feel this is less "real" than the towns themselves. Nonetheless, relations between things, and our understanding of these relations, is crucial for our operation in and interaction with the world. In this sense, mathematics has the form of a language. It must be supposed that our ability to operate with concepts, with relations, had and maybe continues to have an evolutionary value.

It is also curious in this respect that the achievements of mathematics are generally held by mathematicians to be the solution of some famous problem. Certainly such a solution will bring to the solver fame and fortune, or at any rate a certain fame within the world of mathematicians. Yet the history of mathematics and its applications shows that it is the language, methods and concepts of mathematics which bring its lasting value and everyday use. We have earlier mentioned some examples of this. At a more advanced level, we can say that without this language, for example that of groups and of Hilbert spaces, fundamental particle physics would be inconceivable.

Some of the great concepts which have been given rigorous treatments through this mathematisation are:

> number, length, area, volume, rate of change, randomness, computation and computability, symmetry, motion, force, energy, curvature, space, continuity, infinity, deduction.

Very often the problem to make some mathematics is, in the words of a master of new concepts, Alexandre Grothendieck, "to bring new concepts out of the dark". It is these new concepts that make the difficult easy, which show us what has to be done, which lead the way.

More important is the way mathematics deals with and defines concepts, by combining them into mathematical structures. These structures, these patterns, show the relations between concepts and their structural behavior. As said before, the objects of study of mathematics are patterns and structures. These patterns and structures are abstract, a notion discussed below.

What is the methodology of mathematics?

Here again is a subject which is rarely and not widely studied. There is the comment of Paul Erdös that mathematics is a means of turning coffee into theorems. Perhaps, though, this does not help the beginner to much. So let us look at some of the issues discussed in P. Davis and R. Hersh, *The mathematical experience*, and *Descartes' dream*, particularly the section of the first book on "Inner issues". This deals with a number of themes.

(i) *Symbols*

The use of symbols and symbolic notations is one of characteristics of mathematics, and one which puts off the general public. People will say they were able to do mathematics till it got onto x and y. The manipulation of symbols according to rules is still an important part of the craft of mathematics. We find we have to teach people who wish to master say economics but who are unable to deduce from $x+2=4$ that $x=2$. This makes very difficult the understanding of the concepts of economics. Very complicated relations can be expressed symbolically in a way which can hardly be conveyed in words. This economy which symbols allow is improving continually, as the symbols are used in the denotation of advanced concepts and the rules of the symbol manipulation are used to model the rules of the concepts.

It has been said, in an exaggerated way, that the history of mathematics is the history of improved notation. This reflects the nature of intelligence, which requires props and metaphors to help and guide it.

Some symbols are in themselves metaphors. Examples are $<$, $>$, $\leq$, $\geq$, $=$, $\rightarrow$, $\neq$, $\perp$, $\subseteq$, $/$, $\emptyset$, $\int$, $\angle$, and so on. Others have acquired strong associations, so that we can use them as metaphors. Symbols are able to express "with economy and precision", to use words of A N Whitehead. The use of particular symbols is something that changes with time, as mathematicians become accustomed and find appropriate a new notation.

In some cases, a notation, brought about be the laziness of mathematicians, leads to a new theory. For example, expressions of the type

$$(a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n, \dots, a_{m1}x_1+a_{m2}x_2+\dots+a_{mn}x_n)$$

get abbreviated over time to

$$Ax$$

and to allow for the correct manipulation of this abbreviation, the rules for matrices are worked out.

To give an example close to the heart of some of our research, the first author has been concerned for many years as to whether the linear notation for mathematics is a necessity or a historical result, based on the needs of printing. The analysis of this linguistic point has led to a new kind of "higher dimensional algebra", in which symbols are related not just to those to the left and those to the right, but also up and down, or out of the page, as well. This algebra then becomes closer to and more able to model some geometric situations, and this leads to the formulation and proofs of new theorems, to new calculations and insights.

(ii) *Abstraction*

This is an essential part of mathematics, and again is one part of what makes mathematics incomprehensible to the general public.

As said above, mathematical structures are abstract. They are defined by the relations within them. They are thought of as non-sensual. The advantages of abstraction are at least threefold,

(a) An abstract theory codifies our knowledge about a number of examples, and so makes it easier to learn their common features. Only one theory is needed, to replace a multiplicity. This codification exploits analogies, not between things themselves, but between the behavior and relations of things. Finding these analogies, the abstract theory which replaces a multiplicity, is an important method in mathematics.

(b) Once the theory is available, it may be found to apply to new examples. This leads to the excitement and joy of "That reminds me of ...!". For this new example, a body of established theory is available, at the turn of a page.

(c) An abstract theory allows for simpler proofs. This is a surprise, but is commonly found to be true. The abstract theory allows for the distillation of essentials. It is of interest to know if a theorem or fact is true in the general situation or only in the particular example. The abstract theory allows for the removal of possibly irrelevant aspects.

(iii) *Generalization and extension*

This has some features in common with abstraction, but usually applies differently. Thus a generalization of the (3, 4, 5) right angled triangle is Pythagoras' Theorem, while an extension is Fermat's Last Theorem, possibly now settled, which says that the equation $x^n + y^n = z^n$ has no solutions for positive integers x, y and z if $n \geq 3$.

(iv) *Proof*

The rigorousness of the notion of proof is a particular feature of mathematics. It is why mathematics is essential in engineering, safety, physics and so on.

The notion of proof, of validity, in mathematics is an aspect of the general question: What is the notion of validity in an area of study? Each area, from social sciences, economics, chemistry, biology, education, law, literature, and so on, has its notion of validity, and the contrast and uses of this notion are of particular interest.

The question of what is acceptable as a valid argument in mathematics is still subject to argument and discussion, particularly with the existence of very long proofs (for example 15,000 pages), and with the use of computers for visualization, experimentation, and calculation.

(v) *Existence of mathematical objects*

A great mathematician has urged that the major problem of mathematical education is to teach the reality of mathematical objects. What is this reality? In what way do these objects exists?

This question has been a matter of major interest to many philosophers of mathematics, but its interest is perhaps in the process of being downgraded. Mathematics is often about processes. The question of existence of a mathematical structure is maybe like asking whether the game of chess exists. Clearly it does not exist in the way that tables and chairs exist, but none the less, it influences many lives, and passes the cash test. (Does it earn money? The answer is clearly: Yes, for some, for example world champions and makers of chess equipment.)

The relation of mathematical concepts and methods to processes is indicated by the way that muscular memory and rhythm are important aspects of mathematical work. A lot of mathematicians is concerned with the realization and understanding of the effect of repetitive processes and methods.

(iv) *Infinity*

The taming of the infinite, or the enlargement of the imagination to include infinite operations, is one of the joys of mathematics, and also one of the scandals. Are these infinite objects real? The surprise is that these infinite, possibly unreal, objects can be used to prove finite real things, and this again is an aspect of the mystery of the subject. Suppose for example that these infinite objects are used to prove the safety of a nuclear installation, or of an aircraft landing system? What credence should be placed on such a proof? These are real issues.

Is there research going on in mathematics?

Those who wish a practical test should look at the change in Mathematical Reviews since it was started in 1940. This contains abstracts of mathematical papers. Roughly speaking, a few paragraphs are enough for a five page paper. The growth in terms of numbers of pages over these years is about eleven times. Each month now there are published about 400 large pages of abstracts of mathematical papers.

The aims of this research are at various levels. One is the advancement in knowledge about particular types of structures, which are already well defined. Another is the introduction of the study of new structures, as they have appeared and been shown to be relevant. There are new relations between structures. There is the urge to simplification, to find structures which explain structures, and help us understand the way particular structures behave themselves and relate to other structures.

This is indeed the golden age of mathematics.

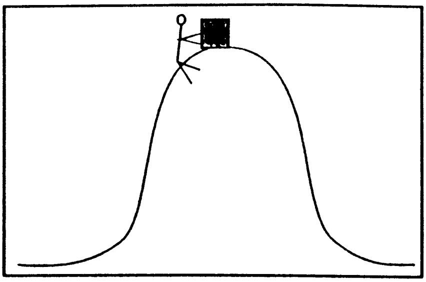
What is difficult for the newcomer in the field, and for the general public, to understand is how one goes about doing mathematical research. Here are a few pointers; we suggest four ways of going about the job, of intention. There are certainly many more, and each individual researcher must in the end devise his or her own strategy for success. It is also difficult to know how much one must know before starting on mathematical research. A famous answer to this particular questions was: "Everything, or nothing".

Method 1: Apply a standard method to a standard type of problem.

This has the guarantee of success, provided one is sufficiently skilful in the standard method. This method is probably a part of every successful research project. Indeed, a common method of mathematical research is to reduce a problem to one already considered. If the original problem is too difficult, then a standard strategy is to simplify the problem so that it does become of standard type, before adding the complications which make it a new problem. The general presumption might be that one can only do easy things. So the method is to reduce a problem to a type that can be seen to be easy. If in doubt, do the obvious thing first.

Those who practice and become skilful at applying standard methods, may someday find that their skills apply to a problem no one else has considered, and that this leads to new and important results.

Method 2: Attack a problem at the frontiers of knowledge.



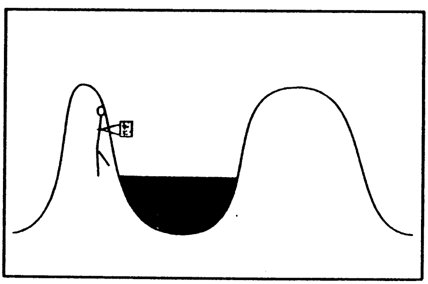
This is the strategy of going for a famous problem at a peak of knowledge. The advantage is that if you succeed, then you will become famous. It is more difficult to assess your chances of success. You will probably need some new ideas.

This seems to be the most ambitious method for a young person. However, S. Ulam in conversation with the first author in 1964 suggested that while this method appeal to a young ambitious person, concentration on this might also not allow for her or him to develop the kind of mathematics most personal characteristic to themselves, because they are solving other peoples' problems.

Usually, though, one attacks smaller problems at the frontiers of knowledge, problems to which not so much effort has been devoted, and so where there is a greater likelihood of success. You will almost certainly have to study to find what has been done, what techniques are available, and which you need to master.

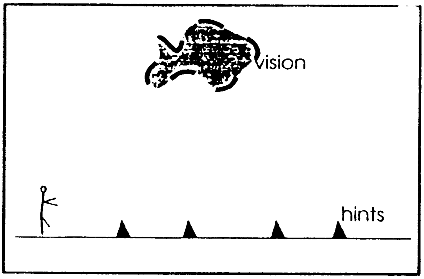
Problems are advantageous if the criteria for success are clear: the answer is yes or no to some question. On the other hand, failure to provide a solution is also clear cut, as is finding the problem too easy. Mathematicians need to build into their strategy plans for dealing with both too little and too much success on the problem at hand.

Method 3: Relate different areas of knowledge.



In this method you learn about the beginnings of different areas, and find relations between them. In this way you fill in the gaps between the peaks, while the "top people" are occupied with building up the peaks. The advantages of this method are that you learn something of different areas, and in a useful way, since you have to work to do the translations between the two areas. This is a good method for PhD theses, since a supervisor can often see the relation without having worked out the details. It also advances the general unity of mathematics. Another advantage is that it gets you used to the idea of proving small but useful results which help to fill in the gaps and create the picture of what is going on.

Method 4: Blue sky research.



Here you have some idea of a mathematics which ought to exist, and the characteristics of it. You also have a few hints as to the kind of materials of which the mathematics ought to be made. The problem is that proper mathematics requires definitions, examples, propositions, theorems, proofs, calculations, and in the beginning none of these exist. So they have to be assembled over a period of time. In what order should this be done, and how important will the work be? This can hardly be judged till the theory is worked out, and such a theory does not emerge, like Venus Anadyamene, fully formed from the sea. A theory accumulates in a journey over a period of years, and a gut feeling of importance of a line of investigation is necessary to motivate travel on a long road.

We have both been working with this kind of research, as well as other kinds, for decades. The first author formulated the theme of higher dimensional algebra in the mid 1960's. In this algebra, symbols are related not just to those to the left or right on a line, but also to those up or down, or even out of the page. The aim was that of an algebra more closely related to the geometry, and allowing a more general type of composition. The expectation was that this algebra would yield some formulations and proofs of new theorems, which would automatically lead to new methods of calculation.

This in the end has proved right, with a lot of people joining in the project. For a long time, though, for example five years, all that could be said was that it was possible to draw pictures which suggested that the ideas would have to work. The problem was a lack of framework to express the algebra corresponding to the pictures, to the geometry. This framework was built up gradually, and it became ever more amazing to see how natural and fitting way it was, once the ideas were thought about in the "correct" manner. Thus, as suggested by Wigner in the quotation giver earlier, the aesthetic criteria for a proper theory were nicely satisfied, and the theory became better than the vision which had prompted it.

It has to be said that, paradoxically, the secret of success in research is the successful management of failure. For if you never fail, then it is likely that the tasks you have set yourself are simply to easy. The interesting research must have an element of risk. You need strategies for dealing with situations when things go wrong: the problem may have proved to hard, or too easy. What comes next? The analysis of the reasons for failure, and the comparison of these reasons with the reasons for wanting to do this problem in the first place, becomes instructive for future work.

What is good mathematics?

We would not like to attempt to give any final answer to this, but all of us should try and formulate some of the aspects that we are looking for. Indeed, as editors of journals, we have to make judgements on this question on a daily basis. For a new mathematical paper we ask: Are the results new? How far ahead do they go of the current literature? Is the paper clean and well written? Is there a clear familiarity by the author with current work in the field, and the relation of his or her results to the field? How surprising are the results? How elegant are the methods? Are there any new methods introduced?

Some of what we call the best mathematics is that which introduces new ideas and concepts which make the previously difficult easy. This contradicts an impression you may have that mathematics is meant to be hard, and is good for you partly for that reason, like a cold bath. To the contrary, good mathematics can, perhaps should, be easy. It is just that often we do not know how to do this. The combination of apparently simple arguments with a surprising conclusion, perhaps with a surprising twist, is what we like best of all.

Conclusion

There is a view that there is no more basic mathematics to be found. This view is comparable to the view of those who have said that physics has ended, the basic problems having been solved.

We feel to the contrary that mathematics is undergoing a revolution, a quiet one, but a revolution none the less. This is occurring on two fronts.

There is first the computational revolution. For computation with numbers, or for graphics presentation, this revolution is well known. Less well known publicly is the computer software which can manipulate symbols and axioms, and other software which can carry out automated reasoning. In principle, these should give mathematicians power to calculate and reason a millionfold more than they can at present, and to deal with the complexities of systems thought previously to be intractable. The prospective effect of these on the teaching of mathematics has yet to be properly understood and assessed, although a lot of work is in progress. The effect on research has already been considerable and is likely to grow in its influence.

A more subtle revolution is the conceptual one. The emphasis on mathematics as the study of structures is finding its mathematicisation in category theory, the mathematical and algebraic study of structures. Category theory has revealed new approaches to the basic concepts of mathematics, such as logic and set theory, and indeed has made respectable the idea that the practice of mathematics needs not one foundation, as traditionally sought, but alternative environments, and a framework for their comparison. These ideas are also important for the progress of computer science, as for example in showing new approaches to data structures.

One of the pleasures of mathematics is the way it operates on various levels, which then interact. So the algebraic study of mathematical structures has itself led to new mathematical structures. Some of these structures have had notable applications in mathematics and physics.

There are still many current dangers for mathematics.

There is a general lack of appreciation of what mathematicians have accomplished, and the importance of mathematics. Some of this has come about through mathematicians themselves, failing to define and explain their subject in a global sense to their students, to the public, and to government and industry. It is possible for a student to get a good degree in mathematics without any awareness that research is going on in the subject

Another is the growing reliance on computers as a black box to give the answer, without understanding of the processes involved, or of the concepts which are intended to be manipulated. So both the scope and the limitations of the computer fail to be understood, the mathematical basis is neglected and perhaps fails to be developed, and the computer may be used in ways which are inappropriate, or simply limited by the software design. It is said that some engineering firms are dispensing with their mathematical research departments in favor of engineers manipulating software packages.

If these dangers are to be averted, then an increased understanding and appreciation of the questions with which we started are essential.

There may be ways of speeding up the process of transfer from the conceptual foresight og the mathematician to the realization in a scientific or technological application. To find them, we need in society a real understanding of the work of mathematicians, and the way mathematics has played a role in the society in which we live. It is our responsibility to the subject we love to find ways of developing this understanding.

Acknowledgements

We would like to thank all those who have contributed to the ideas behind this article. Many of the questions were discussed with students of the final year "Maths in Context" course we ran together, and also with students in the first year course "Ideas in Maths", which ran for students in the Arts Faculty, and also has run now for its second year with first years honors mathematics students. The contributions of these students through discussions and essays have strongly influenced our thinking. Similar questions have been put by the first author to: a British Mathematical Colloquium evening discussion (St. Andrews, 1987); to the Mathematics Department at Aberdeen (January, 1993), with support from Enterprise in Higher Education; to staff and students of the Departmento Geometria e Topologia at the University of Seville (November, 1993); and by the second author to: the session on "Mathematics in Context" at the Conference on "Teaching and Learning Mathematics in Higher Education", at the Oxford Centre for Staff Development, Oxford Polytechnic (1991); in a discussion on "Aims and objectives for a Mathematics Degree" at King's College, London (1992); and by both of us in discussions with Roger Bowers and Brian Denton who run a course on "Mathematics in Society" at Liverpool University. We would like to thank Professor J. H. van Lint for information on the history of coding and CD players.

Ronald Brown and Timothy Porter
Department of Mathematics
University of Wales at Bangor
UNITED KINGDOM

Obituary: Peter Joseph O'Halloran (1931-1994)

Peter Taylor

The death occurred on 25 September 1994 of Peter O'Halloran, founder of the Australian Mathematics Competition and a range of national and international mathematics enrichment activities.

Peter was born in Sydney on 27 April, 1931, the youngest of a family of four boys and three girls. He attended Marist Brothers School in Kogarah, Sydney, and it was there that he developed his life-long fascination with mathematics. He went on to the University of Sydney, where he graduated with a Bachelor of Science and Diploma of Education. After graduation he taught mathematics in several high schools in Sydney and the country regions. In 1965 Peter became head of mathematics at the Royal Australian Navy Academy at Jervis Bay. It was there that he completed a Master of Science, specialising in oceanography.

In 1970, now with four children, Peter and his wife Marjorie moved to Canberra. Peter was one of the original appointments at the Canberra College of Advanced Education (later to become the University of Canberra). At the CCAE Peter developed other interests, including operations research and what was to become his main interest, discrete mathematics.

In 1976, while President of the Canberra Mathematical Association, he established a committee to run a mathematics competition in Canberra. This was so successful that the competition became national by 1978 as the Australian Mathematics Competition, sponsored by the Bank of New South Wales (now Westpac Banking Corporation). It is now well-known that this competition has grown to over 500,000 entries annually, and is probably the biggest mass-participation event in the country.

In 1979 he established the Australian Mathematical Olympiad Committee. The activities of this committee have grown to a complex web of competition and enrichment activities, at the highest level culminating each year in Australia's participation in the International Mathematical Olympiad (IMO).

In 1984 Peter O'Halloran founded the World Federation of National Mathematics Competitions (WFNMC). For several years the main activity of the WFNMC was the production of a Journal, which acted as a vital line of communication for people trying to set up similar activities in other countries. In recent years its activities have expanded to include an international conference and a set of international awards (the Hilbert and Erdös Awards, to recognise mathematicians prominent in enriching mathematics education. Most recently, the WFNMC has become an affiliated Study Group to the International Commission on Mathematical Instruction (ICMI).

Perhaps the most significant event in Peter's career in the last two or three years was his role in the establishment of the Australian Mathematics Trust, which is an umbrella body administering all the activities with which he has been associated and which are referred to above.

Peter's last main duty was to preside at the WFNMC conference in Bulgaria in July 1994. It was obvious to all who were there that Peter was ill. It was generally thought that he was experiencing another bout of pleuro-pneumonia, from which he had suffered in 1993. On his return home however further tests revealed that Peter's condition was much more serious, and cancer was diagnosed. He spent most of his last month at home.

On 31 August he was presented with the David Hilbert Award, which he had declined to accept earlier in the year while still president of the WFNMC. A small party of 30 to 40 of Peter's relatives and local colleagues were in attendance at his home. The David Hilbert Award is the highest international award of the WFNMC and in Peter's case was awarded for "his significant contribution to the enrichment of mathematics learning at an international level".

On 19 September he was awarded the World Cultural Council's "Jose Vasconcelos" World Award for Education at a special ceremony at Chambery, France. This award "is granted to a renowned educator, an authority in the field of teaching or to a legislator of education policies who has a significant influence on the advancement in the scope of culture for mankind".

Peter was also rècognised in many other ways throughout his career. In 1983 he was awarded the Medal of the Order of Australia (OAM), in 1991 he was awarded a Doctor of Science (honoris causa) from Deakin University and in 1994 he was promoted to Professor in his own University.

Many mathematicians have made significant individual contributions to the subject itself. Peter's influence was much more direct, bringing mathematics to the world. With his driving energy and the institutions he created he has significantly increased people's awareness of mathematics and what it can do, throughout the world.

One of Peter's main concerns in life was to assist the disadvantaged. He saw the Australian Mathematics Competition as being able to bring mathematics to children in remote places. In the earliest times of the AMC he travelled the Pacific and introduced the competition to a large number of island nations, such as Fiji, Tonga, Western Samoa, French Polynesia (for whom the paper was made available in French) and many smaller countries, some of which only had radio or occasional steamer contact with the outside world. The Australian Government recognised these efforts and funded this project as one of the few cultural links between Australia and its Pacific neighbours.

Peter saw the main advantage to be derived from the WFNMC as the help it could give to mathematics education in developing countries. I was seated next to him in a debate on the value of competitions at the 1992 International Congress on Mathematical Education (ICME-7) in Québec where he was payed the ultimate compliment to which he would have aspired. One delegate gave a well-planned attack on competitions, based on the usual lines, that competitions encouraged elitism, etc. In response, a delegate from the small African country of Malawi, unknown to Peter, responded with an emotional thank you to Peter and the people of the Australian Mathematics Competition for what they had made possible in her country. This was a most moving experience.

Peter O'Halloran, of course will be irreplaceable. Fortunately, however, he had the foresight to establish institutions in such a way that they all have the resources, particularly human resources, to ensure that the good work will continue.

Peter is survived by his wife Marjorie, four children and six grandchildren.

Peter Taylor,
Acting Executive Director, Australian Mathematical Trust,
Canberra
10 October 1994
(abridged by the Editor)

New President of the WFNMC

The World Federation of National Mathematical Competetions has elected a new president in succession to Professor Peter O'Halloran who died September 1994. The new president is

Professor Blagovest Sendov,
Director, Centre for Informatics and Computer Technology,
Bulgarian Academy of Sciences,
ul. "Acad. G. Bonchev" Blok 8,
1113 Sofia
BULGARIA

ICME-7 Proceedings and Selected Lectures

The *Proceedings of the 7th International Congress on Mathematical Education* (Actes du 7e Congrès international sur l'enseignement des mathématiques), which was held 17-23 August 1992, appeared in May 1994. It is edited by Claude Gaulin, Bernard R. Hodgson, David H. Wheeler, and John C. Egsgard. Another outcome of the congress was the publication of *Selected Lectures from the 7th International Congress on Mathematical Education* (Choix des conférences du 7e Congrès international sur l'enseignement des mathématiques), edited by David Robitaille, David H. Wheeler, and Carolyn Kieran, and also published in May 1994. A set of the two books has been sent to all participants in the congress. Copies of both books may be obtained at a rate of 45,00 Canadian Dollars per book) by contacting the publisher:

Presses de l'Université Laval,
Cité universitaire, Sainte-Foy (Québec), Canada G1K 7P4.
Tel: +1 418 656 5106, Fax: +1 418 656 3305

·Report on II CIBEM, July 1994

Eduardo Luna

After two years of intensive work, on July 17, 1994, at 8:30 p.m., the opening session of the Second Ibero-American Conference on Mathematics Education (II CIBEM) took place. The celebration of the fourth World Soccer Cup won that evening by Brazil did not delay the beginning of this congress that gathered close to 1000 participants representing 18 countries and all Brazilian states.

During this congress several Ibero-American mathematics educators were honored: Benedito Castrucci (Sao Paulo, Brazil), Maria Laura Leite Lopes (Rio de Janeiro, Brazil), Luis Santalo (Buenos Aires, Argentina), Gonzalo Sanchez (Seville, Spain) and Ubiratan D'Ambrosio (Sao Paulo, Brazil).

The activities of each day began with a Plenary Lecture which was transmitted to all Brazil via television the following day. Other activities included: 7 Round Tables, 12 Parallel Lectures, 34 Posters and Projects, Video Sessions, 120 Oral Communications, Working Sessions on 17 different topics, and a Mathematics Fair. In the last mentioned activity, students from local schools exhibited materials related to school projects involving mathematics or its applications to a wide range of topics such as ecology, political issues etc. relevant to the local or national community.

The idea to organize the Ibero-American Congresses on Mathematics Education was proposed by Spanish mathematics educators during the VII Inter-American Conference on Mathematics Education held in Santo Domingo (The Dominican Republic) July 1987. The first of these congresses took place in Seville, Spain, in September 1990. The Universidade Regional de Blumenau, Santa Catarina, Brazil, and the Brazilian Society of Mathematics Education assumed the responsibility to organize the second congress in 1992. The coordinators of the Organizing Committee were Professor Jose Valdir Floriani, Vice-President of Academic Affairs and Professor Maria Salett Biembengut, Department of Mathematics. The quality of the academic and social programs was guaranteed by the hard work of ad-hoc committees that were created by the Organizing Committee.

The academic program was designed by the Scientific Committee, chaired by Professor Ubiratn D'Ambrosio. The Plenary Lectures were the following:

- Inteligencia Multipla: o Lugar da Matematica e da linguagem no aspecto de competencias
Speaker: Professor Nilson Jose Machado, Universidade de Sao Paulo, Sao Paulo, Brazil.

- Etnomatematica - Didactica Fenomenologica - Escuela.
Speaker: Professor Isabel Soto Cornejo, Santiago, Chile.

- O Trabalho de Projeto e a Aprendizagem da Matematica.
Speaker: Professor Paulo Abrantes, Universidade de Coimbra, Lisboa, Portugal.

- Interrogantes en la Ensenanza de las Matematicas.
Speaker: Professor Antonio Perez Jimenez, Universidad de Sevilla, Seville, Spain.

The Proceedings of this congress are being edited by the Organizing Committee and will be published by UNESCO, Paris. This organization also published the Proceedings of the First Ibero-American Congress on Mathematics Education.

The Ibero-American community of mathematics educators are indebted to Universidade Regional de Blumenau, The Brazilian Society of Mathematics Education and to UNESCO for the excellent organization of this scientific event and the publication of its deliberations.

Eduardo Luna
p.t. Barry University
Miami Shores, Florida,
USA

FUTURE CONFERENCES

Regional Collaboration in Mathematics Education: An ICMI Regional Conference, April 1995

This conference, to be held at Monash University, Melbourne, Australia, 19-23 April, 1995, will bring together people from three rather different communities: mathematics and mathematics education, governmental and non governmental organisations, and the computing and telecommunications industry. It will also experiment with the use of telecommunications in linking several off-shore conference sites with the main site at Monash.

The main aim of the conference is to address the issues, problems and mechanisms concerning regional collaboration. The academic content of the conference will be organised around three broad themes: Mathematics and the environment, Teachers and the system, Learners and society, and papers are invited which address these themes in the context of regional development.

Keynote speakers and presenters include: Benvenido Nebres (Phillipines), Cheryl Praeger (Australia), Ken Clements (Australia), Jeremy Kilpatrick (USA), Christine Keitel (Germany), Colette Laborde (France).

The conference is chaired by Professor Alan J. Bishop, Faculty of Education, Monash University. Further information can be obtained from the

Conference Secretariat,
144 Jolimont Road, East Melbourne, 3002
AUSTRALIA,
Tel: +61 3 654 7533
Fax: +61 3 654 8540.

8th SEFI European Seminar on Mathematics in Engineering Education, June 1995

This seminar will be held 28-30 June, 1995, at the Czech Technical University in Prague, the Czech Republic. The aims are 8 (i) to consider cooperation between educational institutions in Europe with a special emphasis on East-West cooperation; (ii) to discuss the role of mathematics in the engineering curriculum with an emphasis on the role of computer packages in engineering education; and (iii) to provide a forum for discussing new technologies in the teaching of mathematics. The programme will include plenary lectures, contributed papers, demonstrations, and a poster session.

For pre-registration (before the end of January 1995), please contact

Fred Simons,
Department of Mathematics,
Eindhoven University of Technology,
P.O. Box 5123,
NL-5600 MB Eindhoven
The NETHERLANDS,

for other information, please contact

Marie Demlová
Katedra matematiky FEL CVUT
Technická 2,
166 27 Praha 6
The CZECH REPUBLIC

47th CIEAEM-Conference, July 1995

The 47th international conference, organised by *La Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM), will take place at the Faculty of Mathematics of the Technical University of Berlin (Germany), 23-29 July 1995. The organising institutions are Freie Universität and Technische Universität, Berlin, in cooperation with Universität Potsdam, Humboldt-Universität Berlin and the Max-Planck-Institute for Educational Research and Human Development, Berlin.

The International Programme Committee and the Local Organising Committee are both chaired by Professor Christine Keitel, Berlin. The main theme of the conference is *Mathematics (education) and common sense: The challenges of social change and technological development*. Key speakers include Professors Philip Davis (USA), Alan Bishop (Australia), Juliana Szendrei (Hungary), and Dr. Rijkje Dekker (The Netherlands). The official languages of the conference are English and French.

Registration no later than 15 January 1995 will be at a fee of DM 150. After this date the fee will be DM 200.

For further information, please contact

Christine Keitel,
Freie Universität Berlin
FB 12, WE 02,
Habelschwerdter Allee 45,
D-14195 Berlin
GERMANY
Tel: +49 30 838 5975
Fax: +49 30 838 5972
e-mail <keitel@zedat.fu-berlin.de>

PDME III, July 1995

PDME III, Political Dimensions of Mathematics Education Conference, will take place in Bergen, Norway, 24–29 July 1995. The official languages of the conference will be English and Spanish. For information on PDME see the article by Stieg Mellin-Olsen, ICMI Bulletin No. 34, page 16–17.

For further information, please contact

Stieg Mellin-Olsen
Institutt for praktisk pedagogikk,
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N-5020 Bergen
NORWAY
Tel: +47 5 544830
Fax: +47 5 544852
e-mail: <mellin-olsen@psych.uib.no>

IX IACME, July-August 1995

The Inter-American Committee on Mathematics Education, and the Chilean Society of Mathematics Education announce that the IX Inter-American Conference on Mathematics Education (IX IACME) will be held at the campus of the Universidad de Santiago, Santiago, Chile, 30 July - 4 August, 1995. On this occasion the Inter-American Committee on Mathematics Education will be celebrating its 35th anniversary.

The main themes of the conference will address the mathematics education problems facing the growth of the Americas: How mathematics education can make a better contribution to the cultural, social and economic development of the Americas. The main activities will be conferences, panels, oral communications, poster sessions, discussion groups and exhibition of materials. Parallel to these main activities there will be special conferences and workshops for mathematics teachers.

The official languages of the Conference will be: Spanish, English and Portuguese. There will be simulataneous translations from Spanish to English and vice-versa during the lectures and panel sessions.

For registration and additional information, please contact:

Fidel Oteiza Morra
Universidad de Santiago de Chile
Casilla 33081, Correo 33
Santiago
CHILE
Tel: +56 681 11 00, ext. 2429

Fax: +56 681 17 39
e-mail: <foteiza@euclides.usach.cl>

SEMT 95, August-September 1995

The International Symposium on Elementary Mathematics Teaching will be held at the Faculty of Education of the Charles University, Prague (the Czech Republic), 28 August-1 September 1995. The programme, which is planned by an international programme committee, will concentrate on the teaching of mathematics to 6-10 year old children. The working language of the symposium will be English. The deadline for abstracts (20 lines) is 20 February 1995.

For further information, please contact

SEMT 95,
Department of Mathematics and Mathematical Education,
Faculty of Education,
Charles University
M.D. Rettigové 4
116 39 Praha 1
The CZECH REPUBLIC
e-mail: <novotna@earn.cvut.cz>

International Conference on Pure and Applied Mathematics, November 1995

A conference of this title will be held at the University of Bahrain, 19-22 November, 1995. For further information, please contact

A.Q.M. Khaliq
Conference Secretary - ICPAM95
Department of Mathematics,
University of Bahrain
P.O. Box 32038, Isa Town,
BAHRAIN

Tel: +973 688348
Fax: +973 682582
e-mail: <icpam95@isa.cc.uob.bh>

1st ACTM, December 1995

The 1st Asian Technology Conference in Mathematics will be held in Singapore 18-21 December 1995. The conference will be hosted by the Association of Mathematics Educators, Singapore, in conjunction with the Nanyang Technical University, National Institute of Education, Singapore, and Radford University, Virginia, USA.

The theme of the conference is *Innovative Use of Technology for Teaching and Research in Mathematics*. The 1st ACTM will provide mathematics educators, computer specialists, technologists, researchers, policy makers, and teachers with an opportunity to share and discuss the latest developments in their areas of specialization. The conference will also provide an avenue for the possibility of collaborative research among participants.

The scientific programme of the conference is planned by an International Programme Committee, chaired by Dr. Wei-Chi Yang, Radford University, Virginia, USA. Papers are invited from people involved in the use of technology in teaching and research in higher institutions and schools. The conference programme will include plenary lectures, paper presentations, and workshops on mathematics teaching and research with the use of technologies. Also an exhibition of educational products with the use of technology will be mounted. The working language will be English.

Abstracts not exceeding 200 words should be sent to

Fong Ho Kheong,
Chair, ACTM 95 Organizing Committee,
Nanyang Technological University,
National Institute of Education,
469 Bukit Timah Road,
Singapore 1025,
Tel: +65 460 5310
Fax: +65 469 8952
e-mail: <fonghk@nievax.nie.ac.sg>

and to

Wei-Chi Yang,
Chair, ACTM 95, IPC
Department of Mathematics and Statistics,
Radford University,
Radford, VA 24142,
USA
Tel: +1 703 831 5332/or 5670
Fax: +1 703 831 6452
e-mail: <wyang@mathstat.ms.runet.edu>

SEACME 7, June 1996

The Seventh South East Asian Conference on Mathematics Education will be held at Hanoi University of Technology, Hanoi, Vietnam, 3-7 June 1996. The organising institutions include the Hanoi University of Technology, the Hanoi Pedagogical Institute No. 1, the Hanoi University, the Research Institute of Education Science, and the Vietnamese Mathematical Society.

The themes of SEACME 7 are *Mathematics education in upper secondary schools*, and *Mathematics education for mathematicians, scientists and engineers, social scientists, and mathematics teachers*. The programme will include invited lectures (delivered by international experts), working groups, topic groups, workshops, national presentations, and posters. Exhibitions of textbooks, software and other types of material are being plannned as well. The conference languages will be English and French.

If you want to obtain the Second Announcement or other type of information, please contact

Nguyen Dinh Tri,
the Organizing Committee of SEACME 7,
Hanoi University of Technology,
Dai Co Viet Road,
Hanoi
VIETNAM

ICME-8, July 1996

The 8th International Congress on Mathematical Education, ICME-8, held under the auspices of ICMI, will take place 14-21 July, 1996, in Sevilla (Spain). The congress will be organised with the Spanish Federation of Societies of Professors of Mathematics (FESPM) as the host organisation. The scientific programme is being planned by an International Programme Committee appointed by the Executive Committee of ICMI. The IPC is chaired by Professor Claudi Alsina, Universitat Politècnica de Catalunya, Barcelona, Spain. The Local Organising Committee is SAEM Thales, Facultad de Matemáticas, Sevilla, while the Professional Conference Organiser is Boréal, S.A, Sevilla.

The congress includes a rich scientific programme which will cover the most important areas in mathematical education. The main activities include plenary and ordinary *lectures*, as well as *working groups*, *topic groups*, *round tables*, *workshops*, *national presentations*, *short presentations* and *projects*. There will also be *exhibitions* of textbooks, software and various other materials. Also the affiliated *Study Groups* of ICMI will contribute to the programme.

For matters concerning the scientific programme, please contact

Claudi Alsina,
Dept. Matemàtiques i Informatica
ETSAB,
Universitat Politècnica de Catalunya
Diagonal 649,
E-08028 Barcelona
SPAIN

To obtain a copy of the First Announcement or further information, please contact

ICME-8
Apartado de Correos 4172
E-41080 Sevilla,
SPAIN

Second European Mathematical Congress, July 1996

This congress will held 21-27 July 1996 in Budapest (Hungary), hosted by the János Bolyai Mathematical Society. The Scientific Committee is chaired by Jürgen Moser, Germany, and the Organising Committee by Gyula Katona, Hungary.

To obtain a copy of the First Announcement, please contact

János Bolyai Mathematical Society,
Fö utca 68,
H-1027 Budapest
HUNGARY,
Tel: +36 1 201 7656
Fax: +36 1 201 6974
e-mail: <h3341sza@ella.hu>

Constructivist Viewpoints for School Teaching and Learning in Mathematics and Science

A new book with the above title was been published in May 1994, with Maija Ahtee and Erkki Pehkonen as editors, by the Department of Teacher Education, University of Helsinki (Research report 131, ISBN 951-45-6733-1, pp. 175).

During the last years in Finland, there have been vivid discussions about the need to improve school teaching. Especially mathematics and science educators have been very active in the discussion about the role and implications of constructivism for teaching. The book is partly based on a specialist seminar on Constructivsm and Mathematics Teaching, held in June 1992 at the Department of Teacher Education in Helsinki. The authors, who include a majority of Finland's leading experts in the field, reflect on what constructivism is and what it means for mathematics and science teaching in Finland. The book is divided into three parts. The first part presents the theoretical considerations which lie behind the reform of school teaching. In the second part, some on-going research projects which are being carried out within the constructivist framework are presented. The last section contains erxamples of action research which active field teachers have carried out in their classes.

The publication may be obtained (at 70 Fmk (about US$ 13) plus postage) from

The Department of Teacher Education,
University of Helsinki,
P.O. Box 38 (Ratakatu 6A),
SF-00014
FINLAND
Tel: +358 0 1918112
Fax: +358 0 1918173

The ICMI Bulletin on E-Mail

The ICMI Bulletin is now stored as an ASCII file in the editor's (i.e. the ICMI Secretary's) electronic post system. If you want to receive a copy of this issue as an ASCII text through e-mail, please contact Mogens Niss at <mn@mmf.ruc.dk>

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