τ-tilting modules for Nakayama algebras

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Notation

Aim

Study support τ-tilting modules for Nakayama algebras

Throughtout this talk,

- K: algebraically closed field
- Λ: basic finite dimensional K-algebra
- $\{e_1, e_2, \cdots, e_n\}$: complete set of primitive orthogonal idempotents of Λ
- $\mathrm{mod}\,\Lambda$: the cat. of fin. gen. right Λ-modules

τ-tilting modules

$M \in \operatorname{mod}\Lambda$

Definition (Iyama-Reiten)

1. M: τ-rigid $:\Leftrightarrow \operatorname{Hom}_\Lambda(M, \tau M) = 0$
2. M: τ-tilting $:\Leftrightarrow$ τ-rigid and $|M| = n (:= |\Lambda|)$
3. M: support τ-tilting Λ-module $:\Leftrightarrow$
   $^\exists e \in \Lambda$: idemp. s.t. M: τ-tilting (Λ/e)-module

   $e \neq 0 \Rightarrow M$: proper support τ-tilting Λ-module

τ: Auslander-Reiten translation

$|M|$: the number of nonisom. indec. summands of M

τ-tilting modules

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Remark

1. M: τ-rigid $\Rightarrow |M| \leq n$
2. tilting $\Rightarrow$ τ-tilting $\Rightarrow$ support τ-tilting
3. Λ:hereditary, tilting $\Leftrightarrow$ τ-tilting

τ-tilting modules

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- $\mathrm{tilt}\,\Lambda$: the isoclasses of basic tilting Λ-modules
- $\tau\text{-}\mathrm{tilt}\,\Lambda$: the isoclasses of basic τ-tilting Λ-modules
- $\mathrm{s}\tau\text{-}\mathrm{tilt}\,\Lambda$: the isoclasses of basic support τ-tilting Λ-modules

τ-tilting modules

Definition-Theorem (Iyama-Reiten)

$M, N \in \mathrm{s}\tau\text{-tilt}\,\Lambda$

- $M \geq N \;:\Leftrightarrow\; \mathrm{Fac}(M) \supset \mathrm{Fac}(N)$
- $\geq$ gives a partial order on $\mathrm{s}\tau\text{-tilt}\,\Lambda$

$$\mathrm{Fac}(M) := \{ X \in \mathrm{mod}\,\Lambda \mid M^n \twoheadrightarrow X \}$$

$\mathcal{K}_\Lambda$: Hasse quiver of $\mathrm{s}\tau\text{-tilt}\,\Lambda$

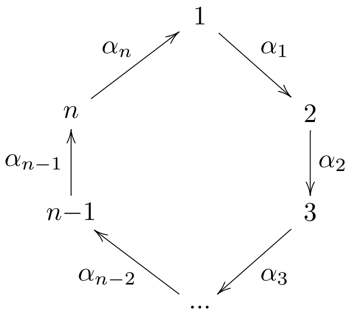
- vertex: $\mathrm{s}\tau\text{-tilt}\,\Lambda$
- arrow: $M \to N \;:\Leftrightarrow\; \begin{cases} M > N \\ \nexists L \in \mathrm{s}\tau\text{-tilt}\,\Lambda \text{ s.t. } M > L > N \end{cases}$

Main result 1
(For self-injective Nakayama algebras)

Self-injective Nakayama algebra

$\Lambda := \Lambda_n^r \simeq KQ/I$: self-injective Nakayama algebra.

Q :



$I = \operatorname{rad}^r KQ$.

automorphism $\phi : \Lambda \to \Lambda$ given by

$$\phi(e_i) = e_{i+1} \text{ and } \phi(\alpha_i) = \alpha_{i+1}$$

Main theorem 1

Let $\Lambda := \Lambda_n^r$ $\quad (\phi : \Lambda \xrightarrow{\sim} \Lambda$ s.t. $\phi(e_i) := e_{i+1})$

Proposition

$\forall$ τ-tilting Λ-module has non-zero projective summands.

Theorem

There exists a bijection between

1. $\tau\text{-tilt}\,\Lambda$
2. $\{M \in s\tau\text{-tilt}\,\Lambda \setminus \tau\text{-tilt}\,\Lambda \mid \operatorname{add} M \cap \operatorname{add} \Lambda = 0\}$

given by

$T = M \oplus e\Lambda \in \tau\text{-tilt}\,\Lambda \mapsto M \in \tau\text{-tilt}(\Lambda/\phi^{-1}(e))$,

$M \in \tau\text{-tilt}(\Lambda/e) \mapsto M \oplus \phi(e)\Lambda \in \tau\text{-tilt}\,\Lambda$

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Main theorem 1

Corollary

Let $\Lambda := \Lambda_n^r$ *and* $r \geq n$.
Theorem induces a bijection

$$\tau\text{-tilt}\,\Lambda \longleftrightarrow \mathrm{s}\tau\text{-tilt}\,\Lambda \setminus \tau\text{-tilt}\,\Lambda$$

In particular,

$$|\,\mathrm{s}\tau\text{-tilt}\,\Lambda| = 2|\,\tau\text{-tilt}\,\Lambda| = 2|\,\mathrm{s}\tau\text{-tilt}\,\Lambda \setminus \tau\text{-tilt}\,\Lambda|$$

Application 1

As an application of Theorem,
we can easily calculate all support τ-tilting Λ_n^r-modules.

How to calculate: (for simplicity, assume $r \geq n$)

(1) Calculate proper support τ-tilting Λ_n^r-modules

$M \in \mathrm{s}\tau\text{-tilt}\,\Lambda_n^r \setminus \tau\text{-tilt}\,\Lambda_n^r$: τ-tilting $(\Lambda_n^r/{}^{\exists}e)$-module

$\Lambda_n^r/{}^{\forall}e$: path algebras of Dynkin quiver of type A

$\tau\text{-tilt}(\Lambda_n^r/e) = \mathrm{tilt}(\Lambda_n^r/e)$

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(2) Calculate τ-tilting Λ_n^r-modules by (1) and Theorem

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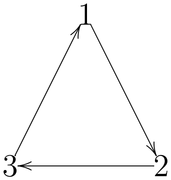
(1) Calculate tilting (Λ_n^r/e)-modules $(^{\forall}e \neq 0)$
(2) Calculate τ-tilting Λ_n^r-modules by (1) and Theorem

We only have to tilting modules for path algebras of Dynkin quiver of type A.

Example: $\Lambda := \Lambda_3^3$

We shall obtain all support τ-tilting Λ-modules

Q_Λ :



$$\Lambda/(e_1+e_2+e_3) = \{0\}$$

$$\Lambda/(e_i+e_j) : \bullet$$

$$\Lambda/e_i : \bullet \longleftarrow \bullet$$

$\begin{smallmatrix}1\\2\end{smallmatrix}\ 2$ $\quad$ $\begin{smallmatrix}2\\3\end{smallmatrix}\ 2$ $\quad$ $3\ \begin{smallmatrix}2\\3\end{smallmatrix}$ $\quad$ $3\ \begin{smallmatrix}3\\1\end{smallmatrix}$ $\quad$ $\begin{smallmatrix}3\\1\end{smallmatrix}\ 1$ $\quad$ $1\ \begin{smallmatrix}1\\2\end{smallmatrix}$

2 $\quad$ 3 $\quad$ 1

0

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(1) Calculate tilting (Λ/e)-modules. $(^\forall e \neq 0)$

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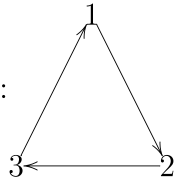

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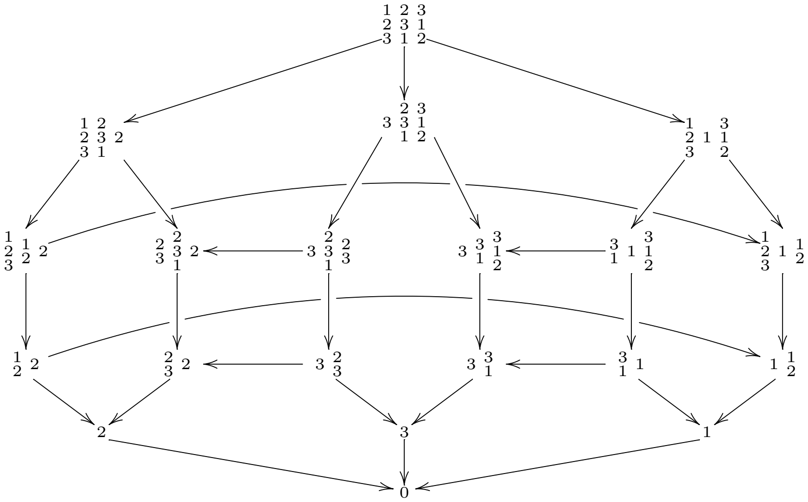

Main result 2
(For Hasse quivers)

Main theorem 2

Λ: fin. dim. K-algebra (not necessarily Nakayama)
Q: indec. projective-injective summand of Λ $(\Lambda = Q \oplus P)$

Lemma (Rejection Lemma of Drozd-Kirichenko)

1. $I := \mathrm{soc}(Q)$: two-sided ideal of Λ
2. $\mathrm{ind}(\Lambda/I) = \mathrm{ind}\,\Lambda \setminus \{Q\}$

Theorem

There exists a surjection

$$s\tau\text{-tilt}\,\Lambda \twoheadrightarrow s\tau\text{-tilt}(\Lambda/I) \quad (M \mapsto M/MI)$$

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$$\mathrm{s}\tau\text{-tilt}\,\Lambda \twoheadrightarrow \mathrm{s}\tau\text{-tilt}(\Lambda/I) \quad (M \mapsto M/MI)$$

- M has no Q as direct summand $\Rightarrow M_\Lambda \mapsto M_{\Lambda/I}$
- $M = Q \oplus U \mapsto (Q/I)_{\Lambda/I} \oplus U$

Application 2

As an application of Theorem,
Λ : Nakayama algebra ($\mathcal{K}_\Lambda$: Hasse quiver of $\mathrm{s}\tau\text{-tilt}\,\Lambda$)
Q: indec. projective-injective summand of Λ ($\Lambda = Q \oplus P$)

Corollary

1. $\ell(Q) > n \implies \mathcal{K}_\Lambda \xrightarrow{\sim} \mathcal{K}_{\Lambda/I}$ $(n := |\Lambda|)$
2. $r > n \implies |\mathrm{s}\tau\text{-tilt}\,\Lambda_n^r| = |\mathrm{s}\tau\text{-tilt}\,\Lambda_n^n|$

Remark

$\ell(Q) \leq n \implies Q \oplus Q/I \oplus U,\ Q/I \oplus U \in \mathrm{s}\tau\text{-tilt}\,\Lambda$

Application 2

Object Level

$$s\tau\text{-tilt}\,\Lambda \longrightarrow s\tau\text{-tilt}(\Lambda/I)$$

$$M \longleftrightarrow M/MI$$

$$Q \oplus Q/I \oplus U \longmapsto Q/I \oplus U$$

$$Q/I \oplus U \longmapsto Q/I \oplus U$$

Application 2

Hasse quiver Level

$$\mathcal{K}_\Lambda \longrightarrow \mathcal{K}_{\Lambda/I}$$

$$M \longleftrightarrow M/MI$$

$$Q \oplus Q/I \oplus U \longmapsto Q/I \oplus U$$

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Application 2

Hasse quiver Level

$$\mathcal{K}_\Lambda \longrightarrow \mathcal{K}_{\Lambda/I}$$

$$\bullet \longleftrightarrow \bullet$$

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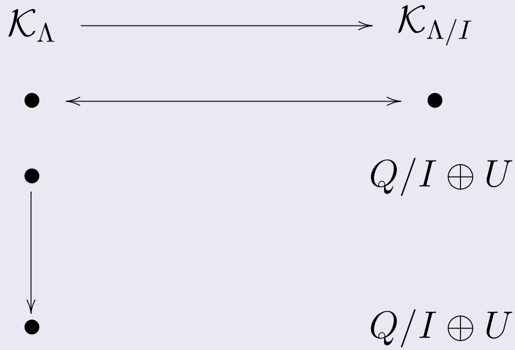

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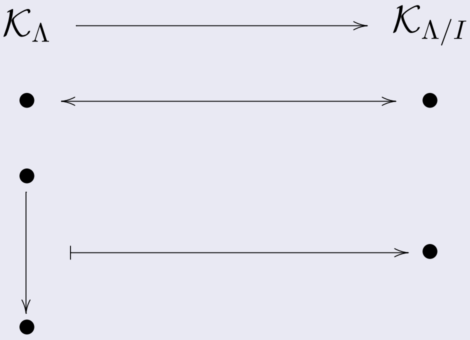

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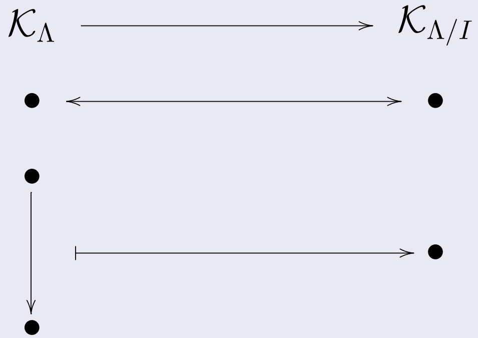
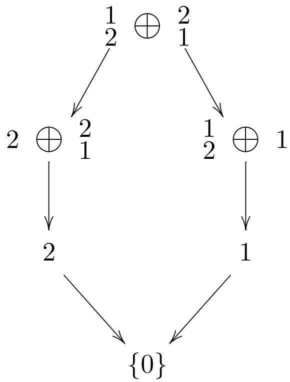
Hasse quiver Level

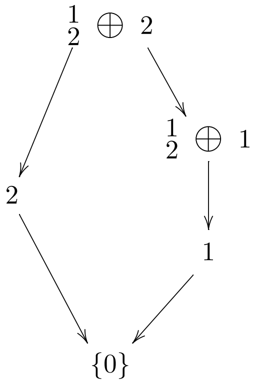


We can combinatorially construct Hasse quivers of Nakayama algebras.

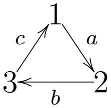
Example

$$\Lambda := 1 \underset{x}{\overset{x}{\rightleftarrows}} 2 \,/ x^2 \qquad\qquad \Lambda/\operatorname{soc}\left(\begin{smallmatrix}2\\1\end{smallmatrix}\right) = 1 \longrightarrow 2$$

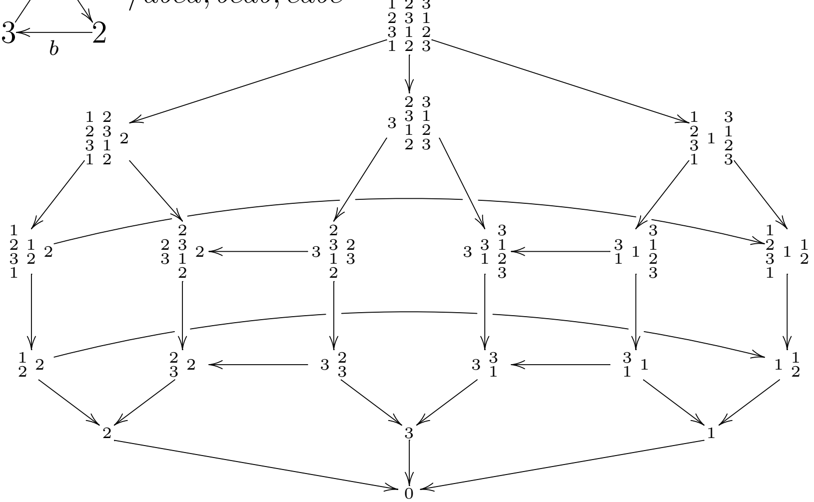




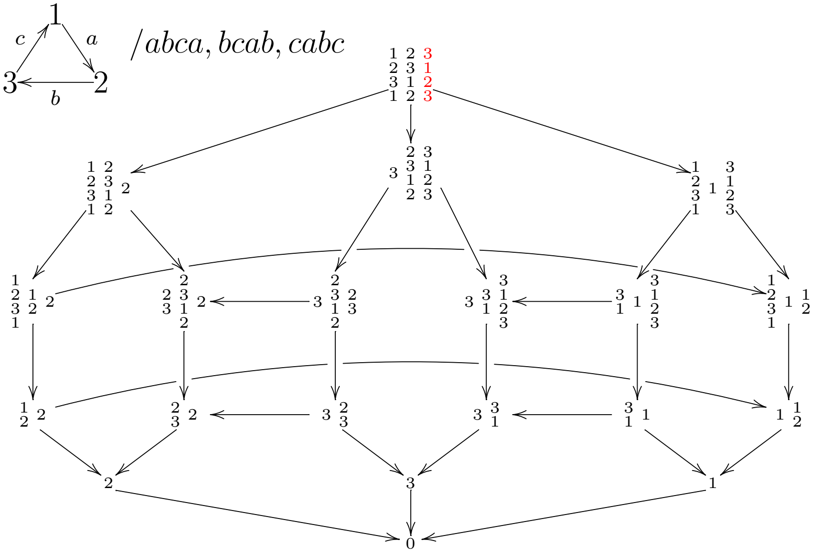
Example

$\Lambda_3^4 =$ 

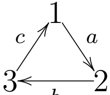
$/abca, bcab, cabc$

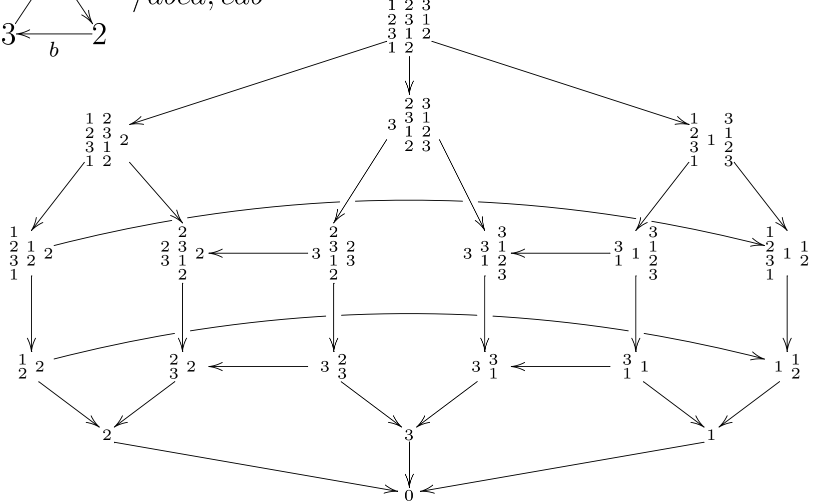


Example

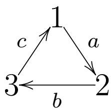
$\Lambda_3^4 =$  $/abca, bcab, cabc$

Example

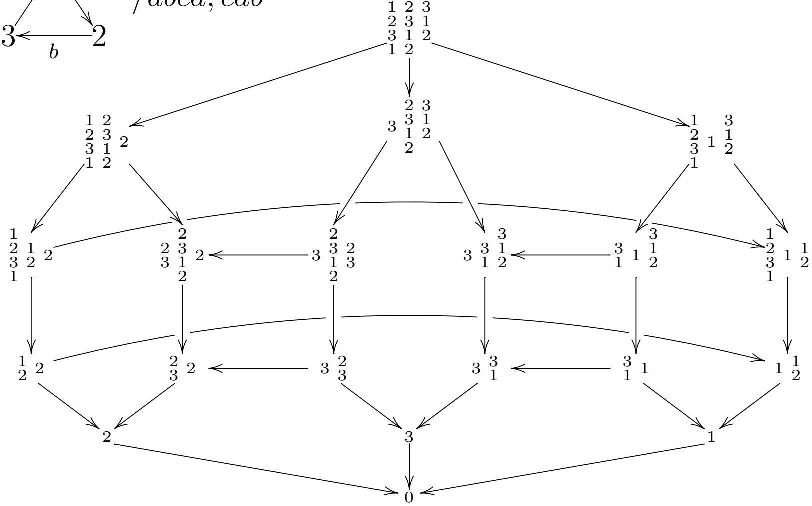
$\Lambda =$  $/abca, cab$



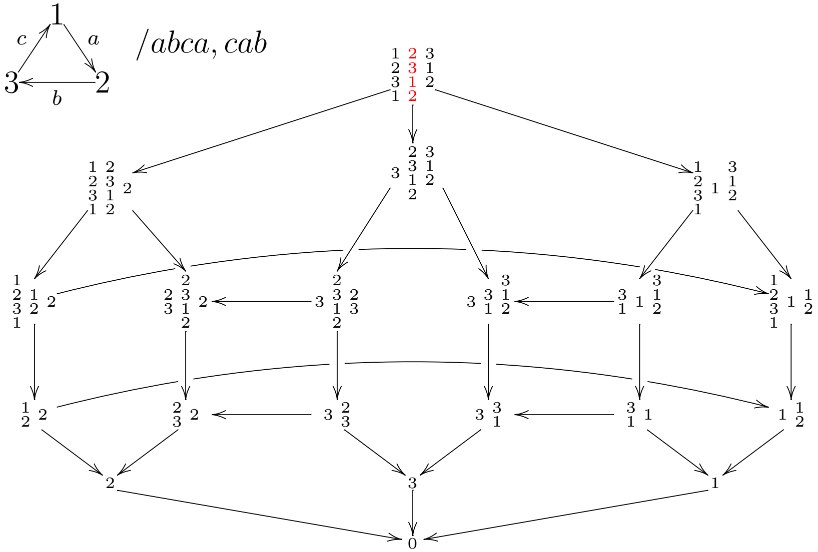
Example

$\Lambda =$ 

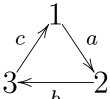
$/abca, cab$



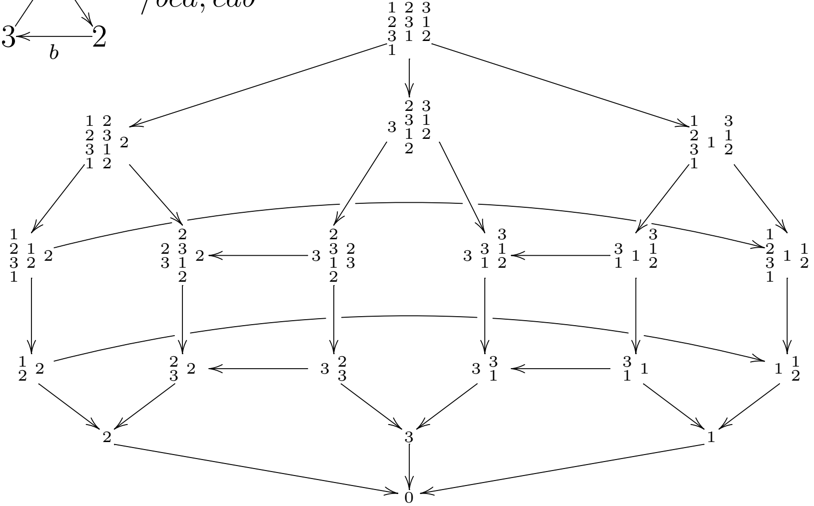
Example

$\Lambda = $  $/abca, cab$

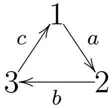
Example

Λ = 

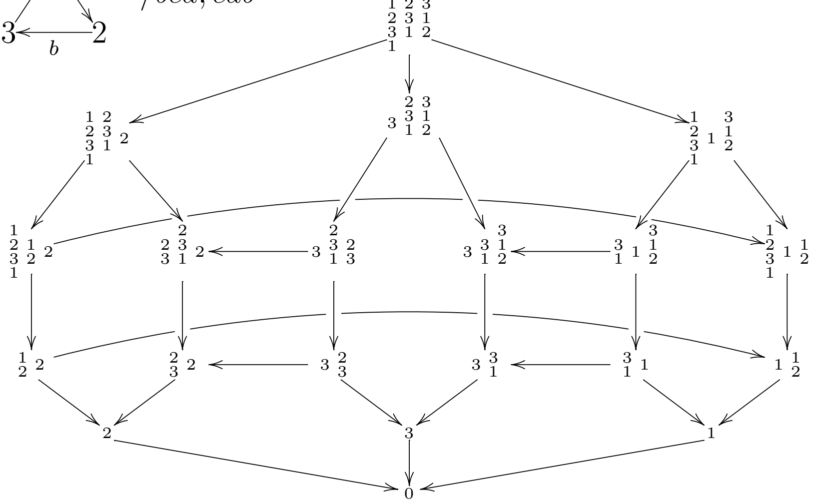
$/bca, cab$



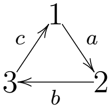
Example

Λ = 

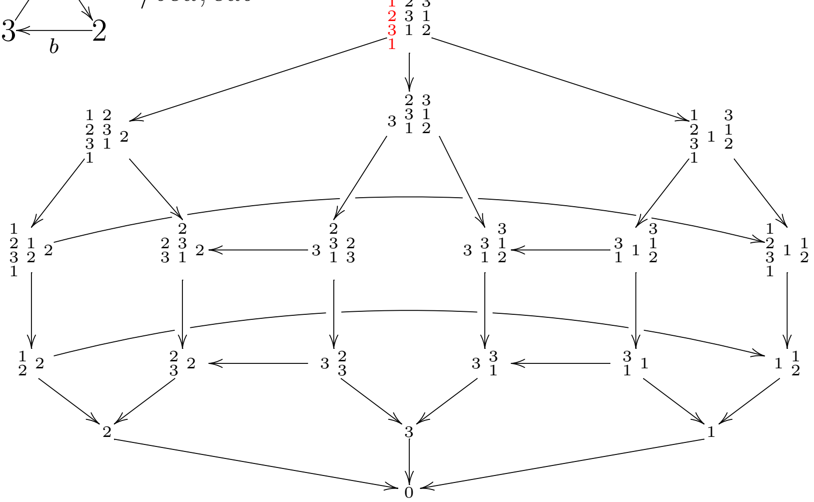
$/bca, cab$



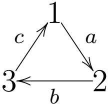
Example

$\Lambda =$ 

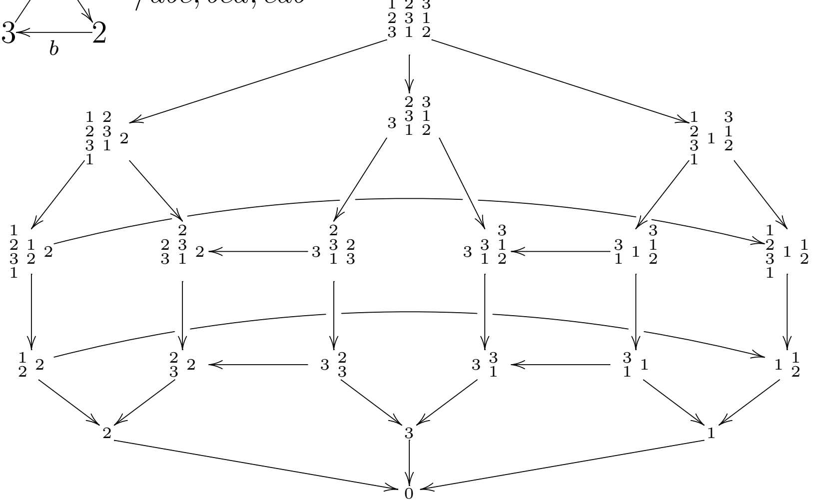
$/bca, cab$



Example

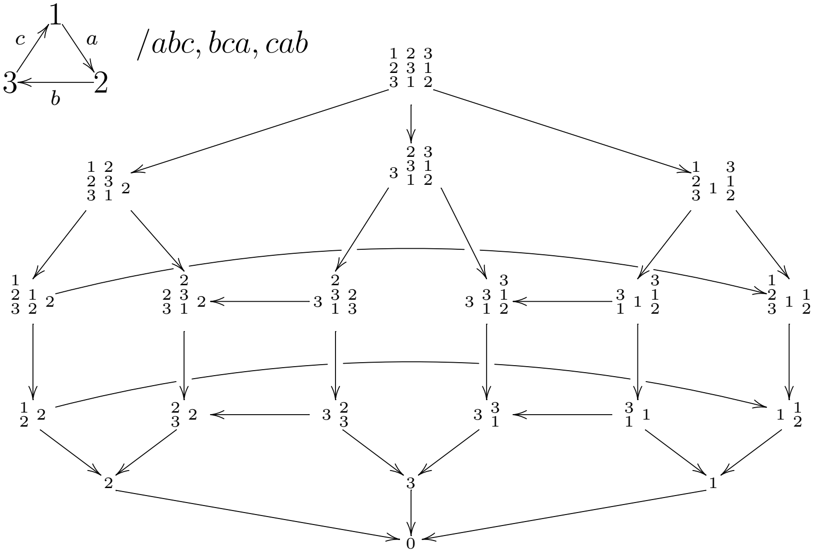
$\Lambda_3^3 =$ 

$/abc, bca, cab$

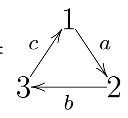


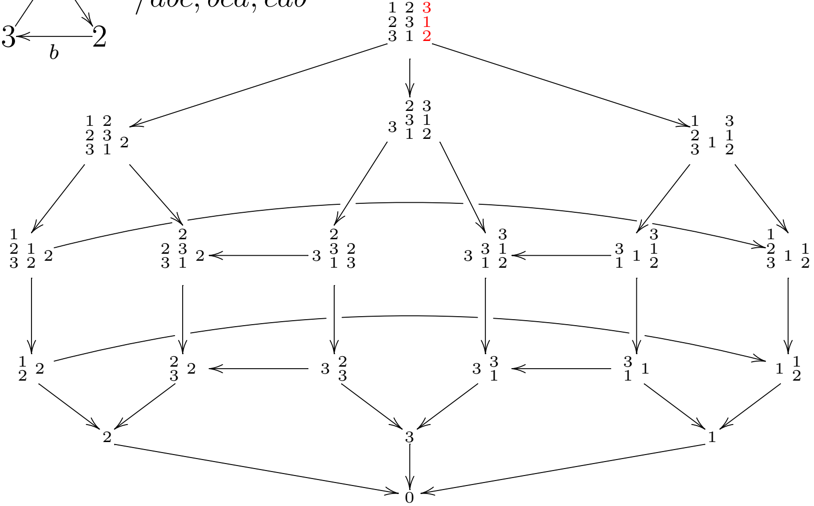
Example

$\Lambda_3^3 = \quad /abc, bca, cab$

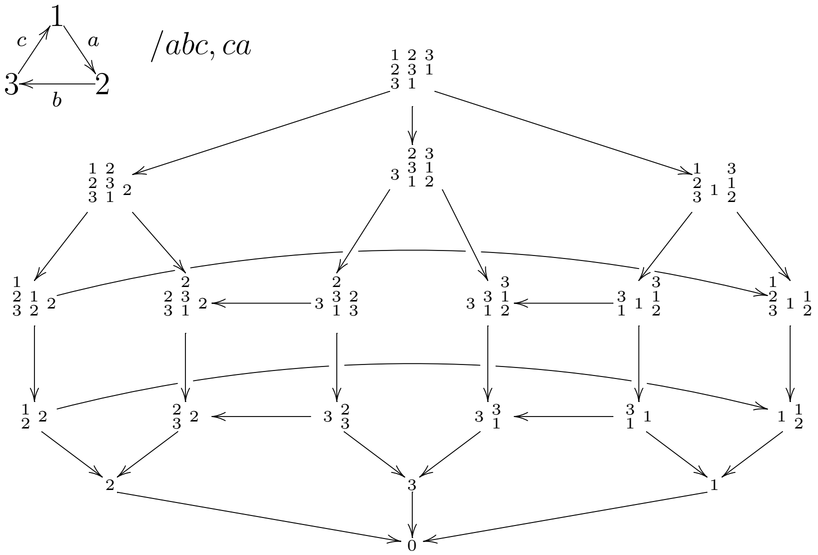


Example

$\Lambda_3^3 =$  $/abc, bca, cab$

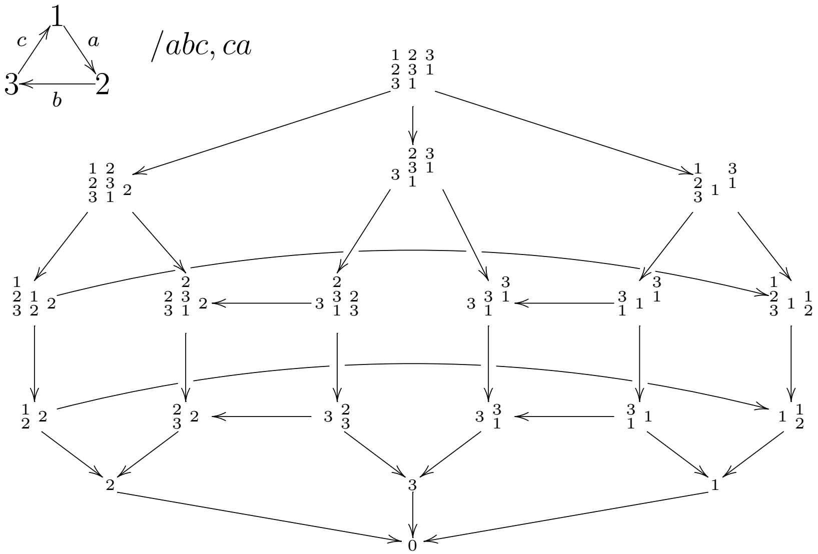


Example

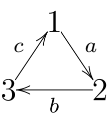
$\Lambda =$  $/abc, ca$

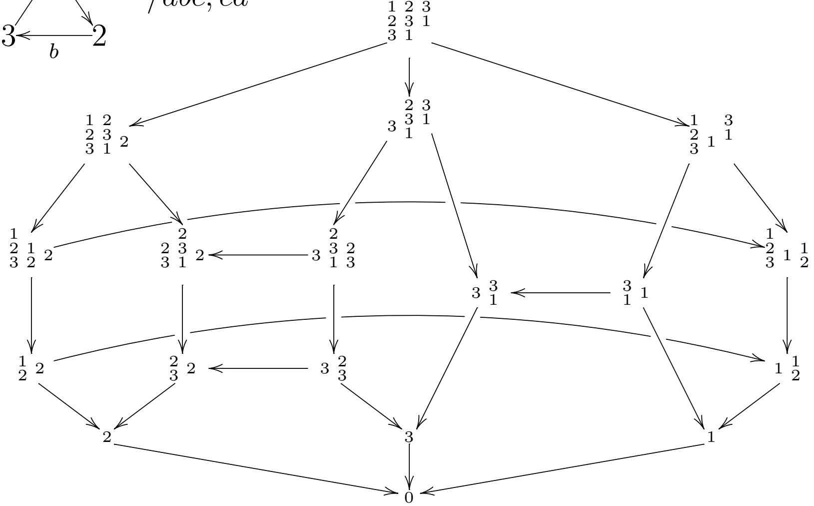
Example

$\Lambda =$ $/ abc, ca$

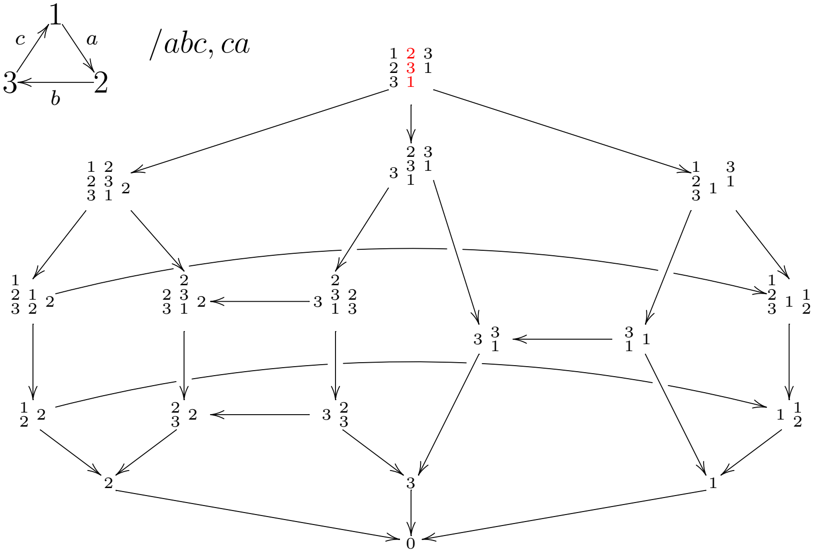


Example

Λ =  $/abc, ca$



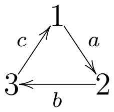
Example

$\Lambda =$ 

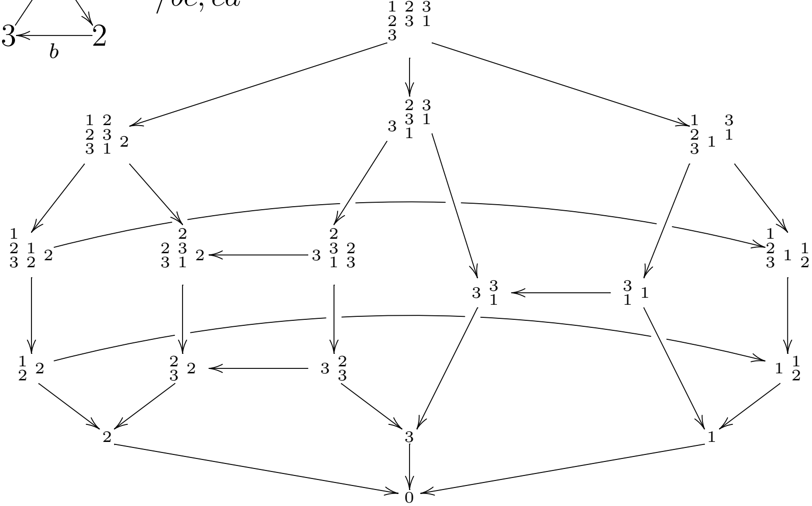
$/abc, ca$

Example

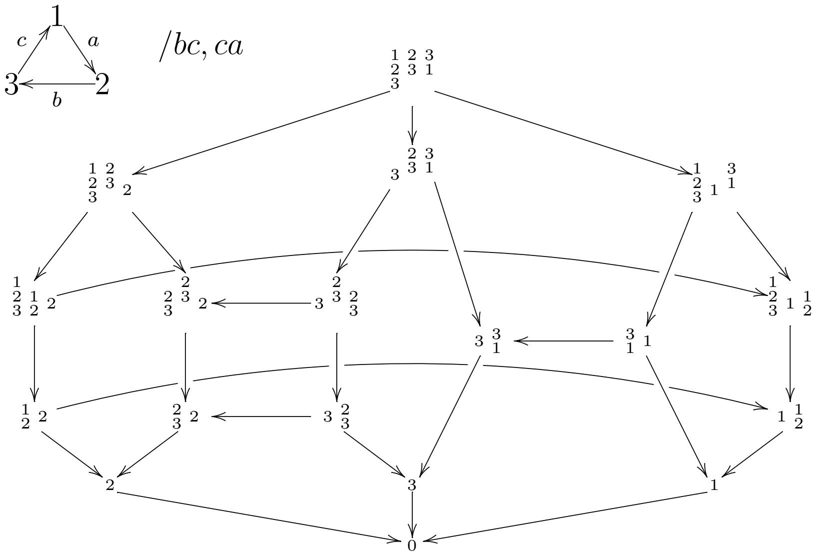
$\Lambda =$



$/bc, ca$

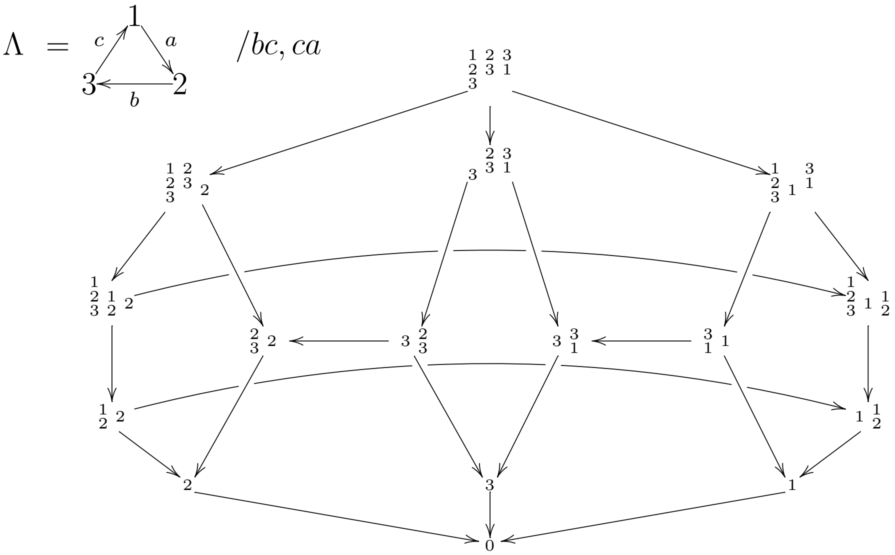


Example

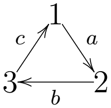
$\Lambda =$ 

$/bc, ca$

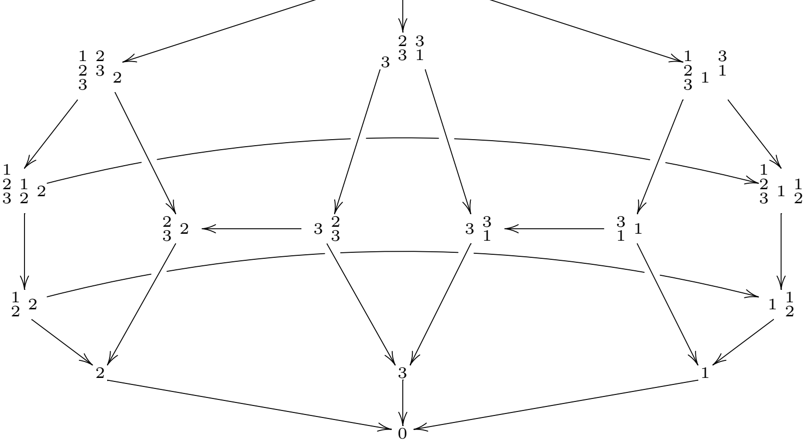
Example



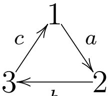
Example

$\Lambda \; = $ 

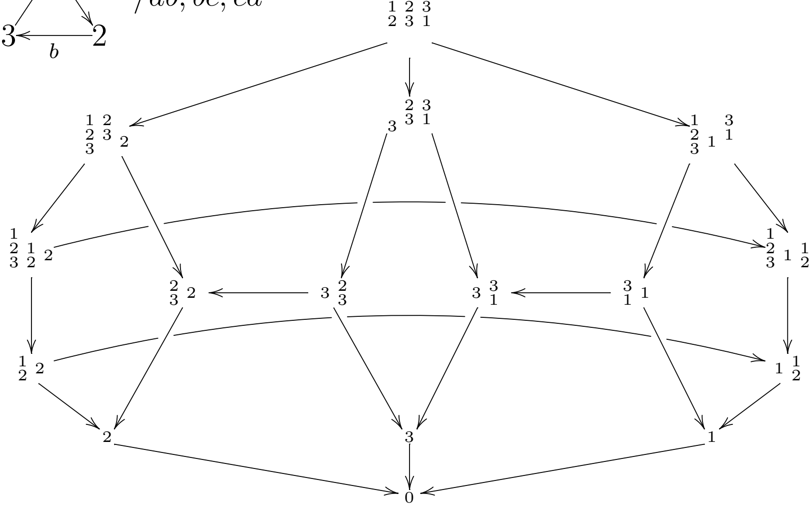
$/ bc, ca$



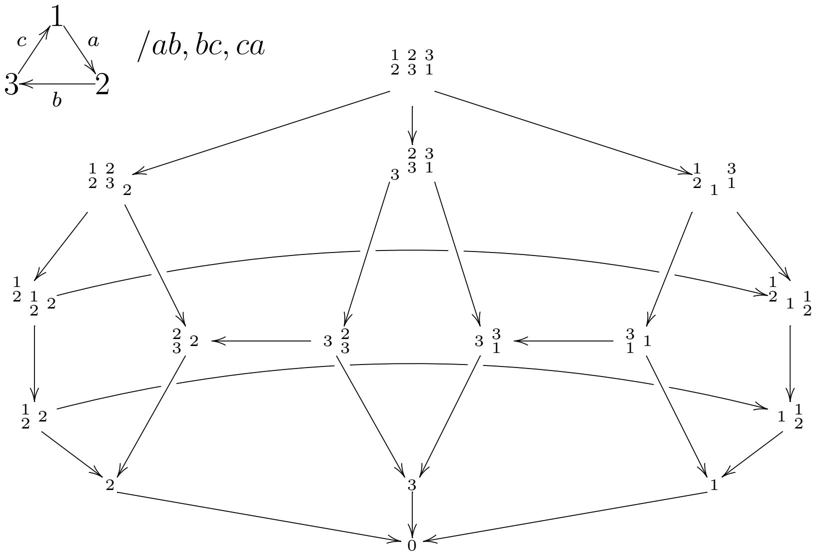
Example

$\Lambda_2^3 =$ 

$/ab, bc, ca$

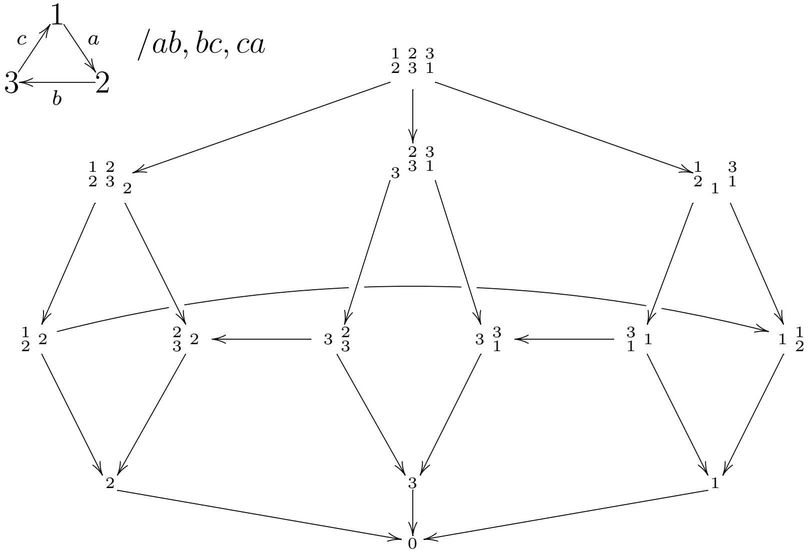


Example

$\Lambda_2^3 =$ 

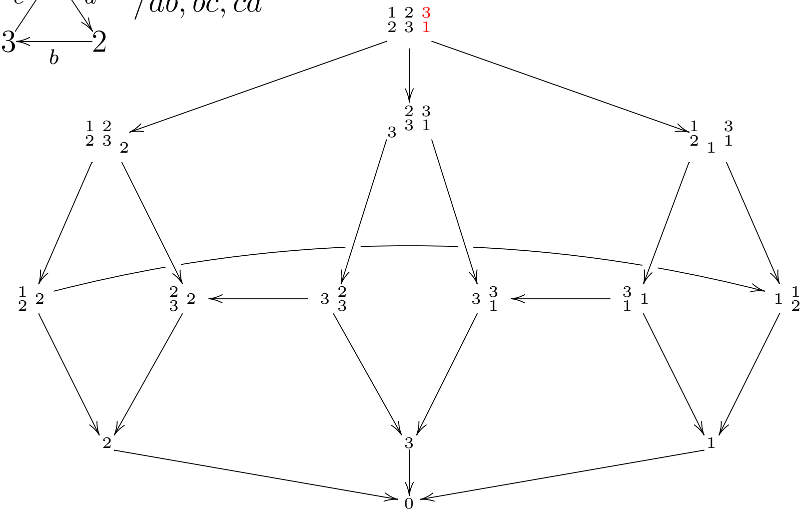
$/ab, bc, ca$

Example

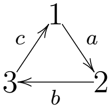
$\Lambda_2^3 =$  $/ab, bc, ca$

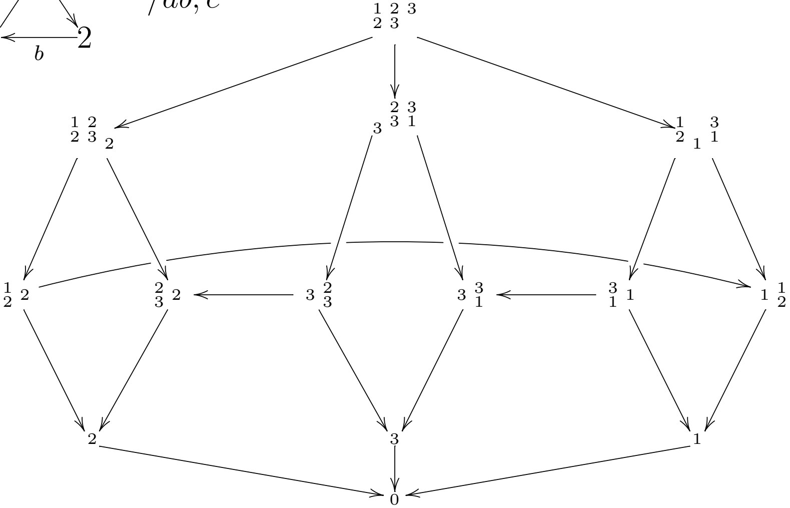
Example

$$\Lambda_2^3 = \quad 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{c} 1 \quad /ab, bc, ca$$

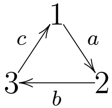


Example

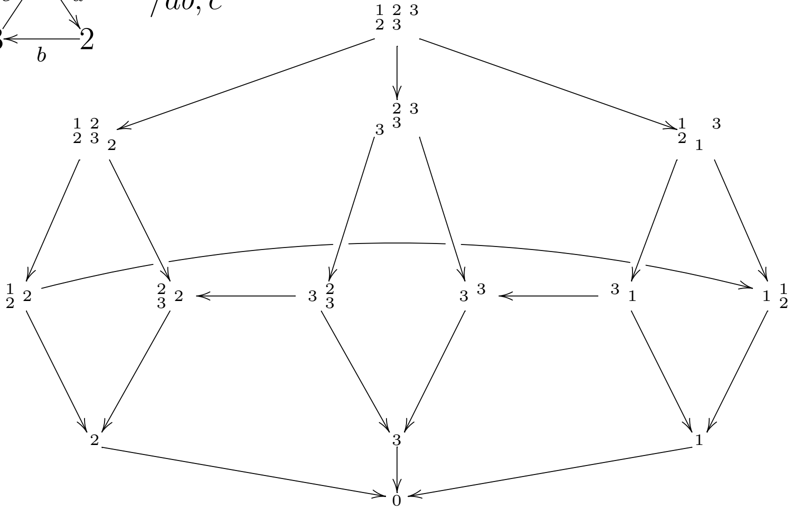
$\Lambda = $  $/ab, c$



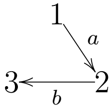
Example

$\Lambda \;=$ 

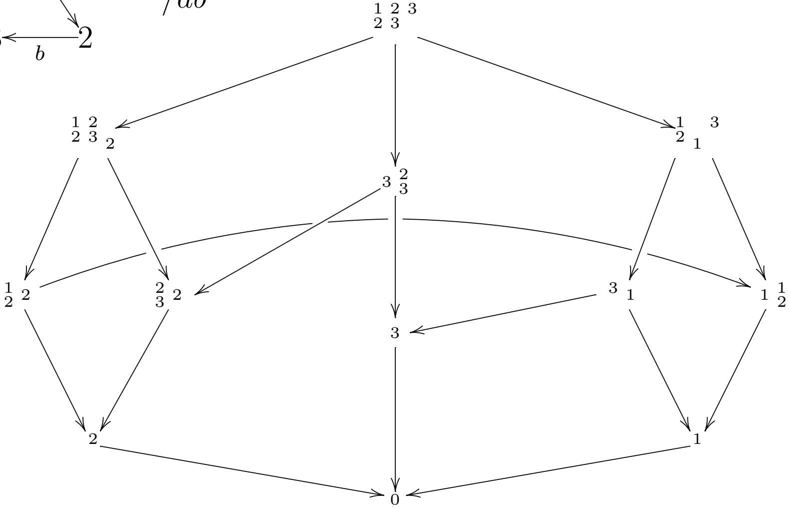
$/ab, c$



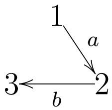
Example

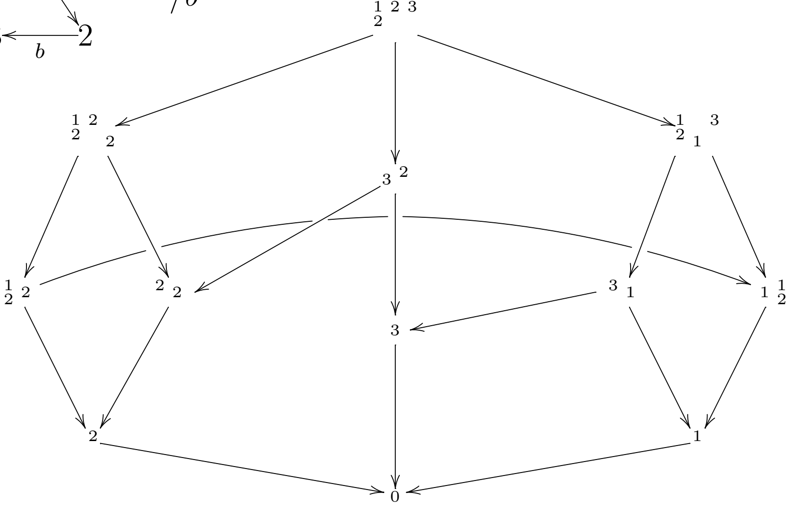
$\Lambda =$ 

$/ab$



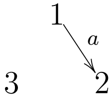
Example

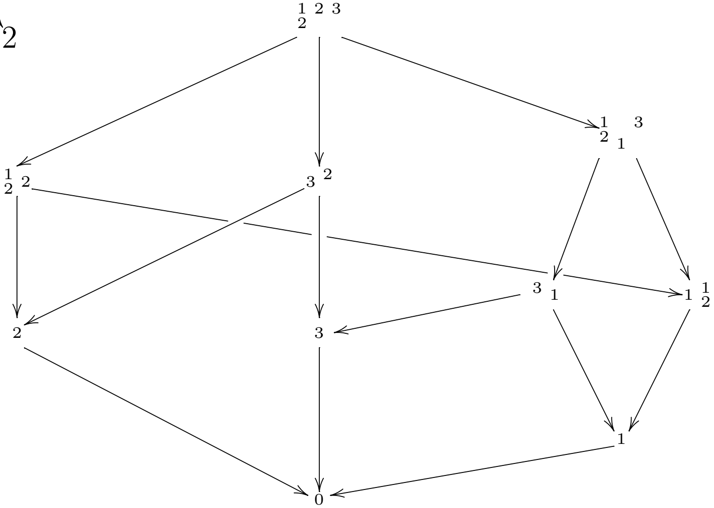
$\Lambda =$  $/b$



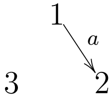
Example

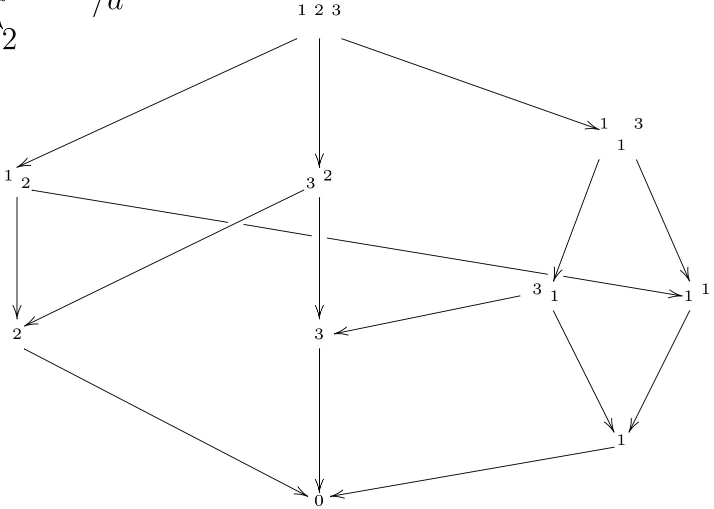
$\Lambda =$





Example

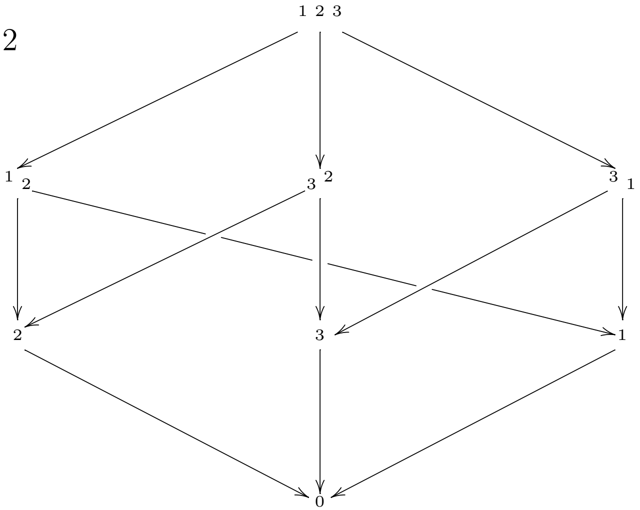
$\Lambda =$  $/a$



Example

$\Lambda \;=\;$ 1

3 2



Application 3

We calculate the nubmber of $\mathrm{s}\tau\text{-tilt}\,\Lambda_n^n$.

n	1	2	3	4	5	6
$\lvert\mathrm{s}\tau\text{-tilt}\,\Lambda_n^n\rvert$	2	6	20	70	252	924

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$$\lvert \mathrm{s}\tau\text{-tilt}\,\Lambda_n^{r\geq n} \rvert = \binom{2n}{n}$$

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$$\lvert \mathrm{s}\tau\text{-tilt}\,\Lambda_n^n \rvert = \lvert \mathrm{s}\tau\text{-tilt}\,\Lambda_n^{n-1} \rvert + \sum_{i=1}^n \lvert \tau\text{-tilt}(\Lambda_n^{n-1}/e_i) \rvert$$

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Fact:

$$\lvert \mathrm{s}\tau\text{-tilt}\,\Lambda_n^{n-1} \rvert = \lvert \mathrm{c}\text{-tilt}\,\mathcal{C}_{D_n} \rvert = \frac{3n-2}{n}\binom{2n-2}{n}, \ \lvert \mathrm{tilt}\,A_n \rvert = \frac{1}{n+1}\binom{2n}{n}$$

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Thank you for your attention!